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**Output gap measurement:  
judgement and uncertainty**

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Jamie Murray

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## Abstract

This paper considers the appropriate definition of the output gap for the purposes of examining the sustainability of the public finances and the uncertainties to which output gap estimates are subject. A range of estimation methods are presented and it is shown that output gap uncertainty is substantial in the UK. Revisions owing to the arrival of new data are on average of the same magnitude as the output gap itself. Uncertainty arising from data revisions is found to make a smaller contribution. Model uncertainty is pervasive. Uncertainty about the output gap carries over to measures of structural borrowing. Since no single estimation method is likely to be reliable at all times, it is suggested that a wide range of evidence should be considered when reaching a judgement about spare capacity and the cyclically adjusted fiscal position.

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# 1 Introduction

- 1.1 The Office for Budget Responsibility (OBR) was established in 2010 and is tasked by Parliament with examining and reporting on the sustainability of the public finances. In reaching a judgement about that sustainability, it is important to consider how the cyclical position of the economy might be affecting revenues and spending. When the economy is operating below its full capacity, elevated unemployment suppresses income tax revenues and boosts spending on out-of-work benefits, for example. Likewise, an overheating economy inflates revenues, since higher wages are needed to tempt more people into the workforce or encourage them to work more hours, and lowers spending on some benefits. The most commonly used measure of spare capacity or overheating is the output gap – the difference between actual output and an estimate of underlying potential output.
- 1.2 Recognising the role played by these cyclical factors, some governments (of which the UK Government is one) aim to achieve balance of a cyclically-adjusted measure of the public finances over a chosen time horizon. In practice, cyclically adjusting the public finances is not a simple task: first, the output gap is not directly observable, is inherently uncertain and is prone to substantial revision; second, even if the cyclical position of the economy could be known with certainty, we would still have to assess the sensitivity of revenues and spending to it. This paper is primarily concerned with the issue of output gap measurement and uncertainty.<sup>1</sup>
- 1.3 This paper begins with a discussion of conceptual issues surrounding the appropriate definition of the output gap for the purpose of assessing fiscal sustainability. The next section describes three sources of output gap revision – data revisions, the arrival of new data and the use of new models. Chapters 3 to 5 illustrate the scale of some of these uncertainties by examining a range of methods, which are summarised in Chapter 6. Chapter 7 considers some implications for cyclical-adjustment of the public finances and Chapter 8 concludes.

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<sup>1</sup> The sensitivity of the public finances to the cycle is the subject of an earlier OBR working paper: Helgadóttir *et al* (2012).

## 2 Conceptual issues

- 2.1 This chapter concerns the appropriate definition of the output gap from the perspective of the fiscal authority (and, by extension, an independent fiscal watchdog tasked with assessing performance against a cyclically adjusted fiscal target), sources of uncertainty and ways to assess the performance of various measures.

### What is the output gap?

- 2.2 The output gap is the difference between actual output and potential output – the maximum level of output that could be achieved while maintaining stable inflation over a given time horizon. It depends on how many people are available to work and how many hours they are willing to put in (labour); the number of buildings, machines and computers that are available to work with (capital); and the efficiency with which they can be combined (productivity).
- 2.3 The formal definition of potential output is thought to have originated at the annual conference of the American Statistical Association, when Okun (1962) described it as the level of macroeconomic output attainable without triggering inflation. He linked this level of output to unemployment via what has come to be known as ‘Okun’s law.’ Of course, the notion that output might deviate from its sustainable level or that employment could fall below full employment has been around for a while – both Keynesian theories of aggregate demand and Wicksellian theories of the neutral interest rate predate Okun’s contribution by decades.

### Time horizon

- 2.4 In both the academic literature and in public discourse, spare capacity is often viewed from the perspective of a central bank, rather than that of a fiscal authority. Normally this is not particularly significant, but there is a distinction between the two that may be more important when the output gap is large. In setting spending plans over the coming years, fiscal authorities are generally interested in what might be considered a long-term measure of spare capacity. This gives an indication of where the level of output might settle once all shocks have worked their way through the economy. Central banks, particularly those pursuing inflation targets, tend to be more concerned with what could be called a medium-term measure of economic slack, that can be expected to influence inflation over their, typically shorter, policy horizons.<sup>2</sup>

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<sup>2</sup> See Box 3.1 of the March 2014 *Economic and fiscal outlook* for a comparison of the OBR output gap and Bank of England spare capacity concepts.

- 2.5 For example, long-term unemployment picked up over the UK's recent recession and Labour Force Survey micro data suggest that the probability of those out of work for at least six months finding a job is significantly lower than that of those who have only recently found themselves unemployed.<sup>3</sup> This suggests that job search intensity is lower among those with longer unemployment durations and, to the extent that this is the case, the long-term unemployed may exert less downward pressure on wages – and so inflation – than the short-term unemployed. So constructing an output gap estimate consistent with an elevated medium-term equilibrium rate of unemployment may improve the accuracy of inflation forecasts.
- 2.6 Over a longer horizon, however, the long-term unemployed are more likely to find their way back into work (absent any structural changes to the functioning of the labour market). Tax receipts will then rise as they begin to pay income tax and welfare spending will fall as fewer people claim benefits. So forecasts of the public finances may be improved by estimating an output gap consistent with the long-term structural rate of unemployment. In summary, when long-term unemployment is high, inflationary pressures could build long before the public finances fully recover, so appropriate definitions of spare capacity and potential output should take account of this.
- 2.7 Many measures of the output gap with a monetary policy emphasis, applied in the academic literature, can be adjusted to take account of this difference in perspective. For example, bottom-up estimates of potential output, such as those derived using a production function, require explicit judgements over the equilibrium rate of unemployment, trend hours worked and activity. And judgements surrounding smoothing parameters, for example, can be adjusted when using statistical filters to produce top-down estimates. In what follows, the methods of output gap estimation are consistent with being from the perspective of the fiscal authority rather than the central bank.

## Uncertainty

- 2.8 The output gap is a notion constructed by economists to help them understand fluctuations in actual output and to formulate policy. It is not something that can be measured directly or known with certainty, even with the benefit of hindsight. Revisions to estimates of the output gap are, therefore, significant. They come from three sources:
- **end-point uncertainty** arises because the future path of output is unknown and it may contain information about the cyclical position of the economy now. This matters more for some estimation methods than others, largely reflecting the assumptions that underpin them and the extent to which information from the future is used to inform current estimates of the output gap;
  - **data uncertainty** arises because the information available at the time is not the final vintage of that data. It is likely to become more accurate with time as more information from that time period becomes available and measurement methods

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<sup>3</sup> Bank of England (2014a)



improve. Some methods are more sensitive to this than others, dependent on the degree to which revisions are attributed to the level of potential output or the output gap; and

- **model uncertainty** reflects mainly changes to our understanding of how the economy functions. Generally, methods with a richer economic structure would be more susceptible to this source of uncertainty but, as our understanding of the process governing the growth of productivity evolves, for example, so might our view on the volatility of potential output – which would affect the estimates from all the methods presented in what follows.

2.9 The susceptibility of output gap estimates to different sources of revision is examined throughout this paper. The effect of new data is assessed by comparing the real-time and ex-post estimates for a variety of the methods presented;<sup>4</sup> the influence of revisions to data is explored in Box 6.2; while model uncertainty is reflected in the wide range of output gap estimates produced using different methods.

## Assessing performance

2.10 Since the potential output of the economy cannot be measured directly, we will never know whether the estimates we construct are accurate. This makes it difficult to assess the benefits of one method of estimation over another. Ideally, we would like a measure that can be calculated accurately in real time (i.e. is not revised much) and appears plausible with hindsight. It is tempting to rank output gap measures only on their tendency to be revised, but it would be easy to win this competition using a method that sets actual output equal to potential output at all times and is never revised. Unfortunately, such a method would be useless for forecasting, since, among other things, it would implicitly assume that the unemployed never return to work. So we need to look beyond just tendency for revision when assessing the performance of measures.

2.11 For central banks, estimates of the output gap can be assessed based on their performance in explaining that element of inflation thought to depend on demand pressures. For fiscal authorities (and fiscal watchdogs), a sensible metric might be the extent to which the output gap explains cyclical variations in the public finances. But there is an obvious circularity here – how do we know the cyclical parts of inflation and the fiscal balance without already having an estimate of the cycle? Insofar as the estimated output gaps that best explain inflation are likely to have been estimated using the Phillips curve, for example, then they will be ranked (possibly undeservedly) top.

2.12 Economists typically consider a wide range of evidence when forming a view on the margin of spare capacity in the economy, supported by a narrative that reflects their subjective interpretation of economic developments. An alternative approach to quantitatively evaluating various output gap measures (and having to place a weight on tendency to

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<sup>4</sup> I use the current vintage of output data available up to a given point in history for this, while a true real-time estimate would make use of earlier vintages of data.

revision relative to theoretical coherence) is to ask whether they are consistent with our broader understanding of economic history. Clearly, a method indicating significant overheating during the Great Depression might have been a rather unhelpful guide to subsequent economic prospects.

- 2.13 In what follows, judgement is applied in considering whether the estimated output gap series 'look' sensible, as well as with an eye to their revisions properties. Box 6.1 provides an example of this approach with respect to the productivity story underlying recent revisions to pre-crisis estimates of the output gap. What each measure has to say about the amplitude of the cycle, the volatility of the output gap and the noisiness of potential output growth is explored in Chapter 6.

### 3 Estimating the output gap: univariate methods

3.1 This section considers univariate methods – those that utilise just the output series itself. In what follows, I choose real, non-oil, gross value added per capita as the measure of actual output.<sup>5</sup> The motivation for this is that the size of the working-age population is unlikely to be closely related to the cyclical position of the economy. Scaling by the population prevents demographic trends from introducing noise to the estimated output gaps. I also exclude the oil sector, which accounts for a tiny percentage of employment but a more significant share of output – oil output is very volatile since it can be affected by maintenance procedures, for example. In what follows, the logs of actual output, potential output and the output gap are given by  $y_t$ ,  $y_t^*$  and  $c_t$  respectively and are related by the identity presented in Equation 1.

$$y_t = y_t^* + c_t \tag{1}$$

#### Linear de-trending

3.2 The simplest method for estimating the path of potential output is to assume it is a straight line. More complicated is deciding the dates between which it should be drawn. Charts 3.1 and 3.2 show the estimates of the output gap associated with drawing a straight line from the first quarter of 1965 and measuring deviations of actual output from it.

Chart 3.1: Linear output gap

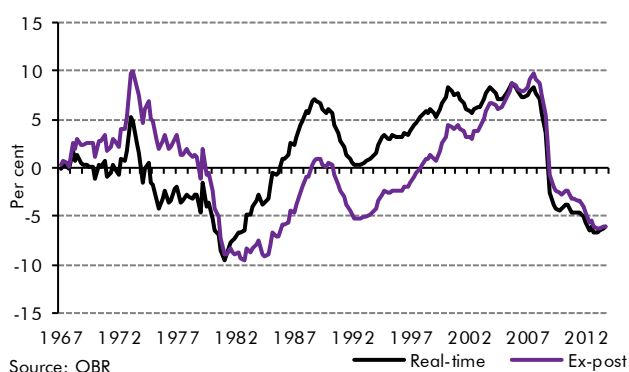
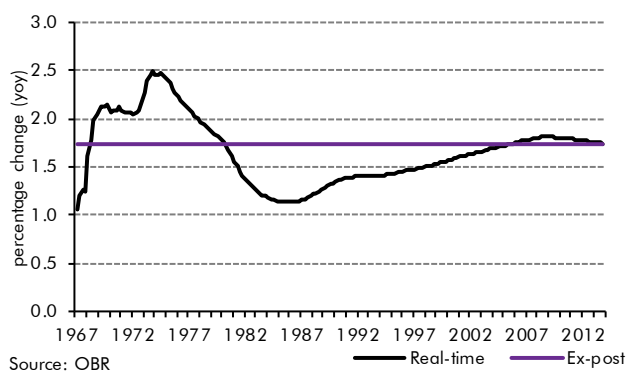


Chart 3.2: Linear potential growth



3.3 The first chart plots estimates of the output gap in real time and ex-post – where real time means using only data up to the quarter in question and ex-post estimates make use of the whole sample.<sup>6</sup> The revisions are shown to be very large – this is because, as time passes,

<sup>5</sup> Where the population is defined as those aged 16 years or over, as recorded by the Labour Force Survey (LFS). Prior to the LFS (which began in 1971), I use annual total population figures, available from the ONS, interpolated to a quarterly frequency and spliced to the later data.

<sup>6</sup> I use the current vintage of output data available up to a given point in history for this, while a true real-time estimate would make use of earlier vintages of data. The difference this makes is the subject of Box 6.2.

we know more about the historical rate of growth and update the slope of our line accordingly (Chart 3.2). Towards the end of the sample, the ex-post estimates (made using the full sample) converge with the real-time estimates of the output gap. This method is relatively sensitive to the chosen starting point – shifting it 5 years forward, for example, reduces the size of output gap in the final quarter of 2013 from -6.4 per cent to -6.0 per cent.

## Hodrick-Prescott (HP) filter

3.4 The HP filter – Hodrick & Prescott (1997) – is based on two beliefs:

- output does not deviate too far from its trend level (cycles are not too big); and
- the growth rate of potential output is relatively smooth (potential output is not too volatile).

$$\sum_{t=1}^T \left( \frac{1}{\sigma_1^2} (c_t)^2 + \frac{1}{\sigma_2^2} (\Delta y_{t+1}^* - \Delta y_t^*)^2 \right) \quad (2)$$

3.5 The filter chooses  $y_t^*$  such that loss function (2) is minimised, where  $\sigma_1^2$  is the variance of the output gap and  $\sigma_2^2$  is the variance of trend growth. The user of the HP filter can specify the relative weight placed on the two beliefs by constraining the ratio of the two variance terms to be equal to a specific value, given by  $\lambda$  – as shown in Equation 3.

$$\lambda = \sigma_1^2 / \sigma_2^2 \quad (3)$$

3.6 It is clear from Equation 2 that penalising the smoothness of potential output is the same as minimising the sum of squared residuals from the equation:

$$\Delta y_{t+1}^* = \Delta y_t^* + \varepsilon_{2,t+1}. \quad (4)$$

3.7 And, in minimising the sum of squared deviations of actual output from trend output, we are minimising the sum of squared residuals from the equation:

$$c_t = \varepsilon_{1,t}. \quad (5)$$

3.8 The HP filter can also be represented in state-space – the signal equation is given by the identity presented in Equation 1, the state equation for potential output is given by Equation 7 (a manipulation of Equation 4) and the state equation for the output gap is given by Equation 8:

$$\text{Signal: } y_t = y_t^* + c_t \quad (6)$$

$$\text{State: } y_{t+1}^* = 2y_t^* - y_{t-1}^* + \left( \frac{\varepsilon_{1,t}}{\lambda^{1/2}} \right) \quad (7)$$

$$\text{State: } c_t = \varepsilon_{1,t} \quad (8)$$

- 3.9 The lambda term has been applied such that the error terms are constrained by the relative weight placed on output gap minimisation and potential output growth smoothness. The state-space representation of the HP filter can be solved using the Kalman algorithm, which provides one-sided and two-sided estimates of the output gap (i.e. real-time and ex-post estimates) – Kalman (1960).
- 3.10 The value represented by the choice of lambda is the only explicit judgement associated with the HP filter and is one subject to significant debate in the literature. The authors, Hodrick and Prescott, posit that a value of 1600 is appropriate for quarterly data reflecting the view that this tallies with their subjective assessment of the US business cycle. Pedersen (2001), for example, argues that a value of 1000 is better. Of course, there is no reason why the UK business cycle must be characterised by the same degree of persistence as in the US or that the business cycles today should typically be of the same length as in the past. And one might reasonably expect the appropriate value of lambda to vary with the time-horizon of the output gap being measured.
- 3.11 While it is the choice of lambda that often gets the most attention, the HP filter is also consistent with a number of other implicit assumptions and judgements:
- the data-generating process governing the evolution of potential output is assumed to be a random walk with stochastic drift;
  - from the above, the best indicator of tomorrow's potential output growth is today's potential output growth;
  - the output gap is an independent and identically distributed random variable, so the best guess of tomorrow's output gap, given today's output gap, is zero (no expected persistence);<sup>7</sup> and
  - shocks to demand are not correlated with shocks to supply.
- 3.12 A number of these implicit judgements are important, but the assumption that demand shocks are uncorrelated with supply is particularly contentious, since it rules out the possibility of periods of protracted cyclical weakness permanently lowering the level of output. Delong & Summers (2012), for example, take a different view – they posit that a large negative output gap can have very persistent effects on the level of potential output, via hysteresis effects in the labour market or reduced investment lowering the capital stock, for example. The IMF (2012), in its advice to the UK government, used an estimate that a

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<sup>7</sup> In practice, persistence of the output gap is introduced via the smoothing parameter – similar results could be obtained by instead assuming the output gap is an autoregressive process and calibrating the parameters of that equation.

negative output gap of 1 per cent might lower the level of potential output by around 0.1 per cent a year and found that much of the effect in the UK is accounted for by labour market hysteresis.

- 3.13 The way in which hysteresis effects influence estimates of the output gap depends on the time horizon of the output gap measure in question. For example, hysteresis effects in the labour market will tend to push up the medium-term equilibrium rate of unemployment, but may affect the long-term structural rate by less. So estimates of the output gap on a medium-term basis would be likely to report less slack in the economy than those aiming to capture a longer-term measure of spare capacity. Hysteresis effects are also important to forecasts of potential output, although that is not the subject of this paper.
- 3.14 At the end of the sample, there are no future data available to assist with estimating the current level of potential output – the so-called end-point problem. All two-sided filters suffer from this problem and are revised once future data become available. Filters respond differently to the end-point problem depending on the assumptions that underpin them. The specific assumptions underpinning the HP filter mean that, at the end of the sample, potential output growth tends to be biased down when the output gap is negative and biased up when it is positive. It is shown later that other filters may respond differently, but do not necessarily offer a more favourable balance of characteristics.

Chart 3.3: HP output gap

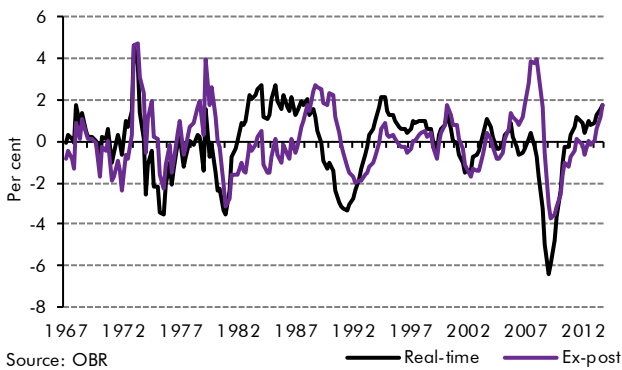
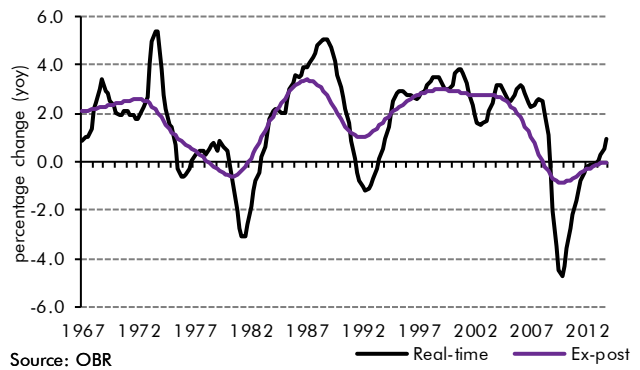


Chart 3.4: HP potential growth



- 3.15 Chart 3.3 illustrates the HP filter estimates of the output gap (with lambda set to 1600) in both real-time and ex-post. Perhaps the most striking thing about this chart is that revisions tend to be largest around recessions and particularly over the most recent recession. In 2007, this measure would have said the economy was operating at around its trend level of output. It now says the economy was overheating by around 4 per cent of potential GDP – and that potential output growth began to slow in the early 2000s.
- 3.16 This specific reinterpretation reflects the loss function of the filter – because sharp movements in potential output growth are heavily penalised, the two-sided filter begins the slowdown ahead of the recession, to avoid making a very large adjustment when the crisis hits. The one-sided (real-time) version of the filter cannot see this coming so is forced to allocate the shocks to potential growth and the output gap as they become apparent. It is

worth considering that the assumptions underpinning the HP filter are consistent with a specific view of the process governing potential output – Box 6.1 describes those views in more detail.

- 3.17 The average revision associated with the HP filter over the time period selected is around 1.4 percentage points (a little larger than its average absolute size of 1.1 per cent). The latest estimate of the output gap provided by the HP filter (i.e. those for the final quarter of 2013) do not change materially when the sample start point is shifted forward five years.
- 3.18 The distribution of actual output is consistent with booms gathering pace gradually but recessions being more abrupt. This real-time asymmetry can partly be ameliorated by placing more weight on estimates of past potential growth than the HP filter does, which can be achieved by altering the objective function of the filter.

## Prior-constrained (PC) filter

- 3.19 Like the HP filter, the prior-constrained (PC) filter<sup>8</sup> is based on two beliefs – the first is the same but the second differs:

- output does not deviate too far from its trend level (cycles are not too big); and
- the growth rate of potential output does not differ too much from its historical average rate of drift.

$$\sum_{t=1}^T \left( \frac{1}{\sigma_1^2} (c_t)^2 + \frac{1}{\sigma_2^2} (\Delta y_{t+1}^* - \text{drift}_t)^2 \right) \quad (9)$$

- 3.20 The PC filter chooses  $y_t^*$  such that the loss function given by (9) is minimised.  $\sigma_1^2$  is the variance of the output gap and  $\sigma_2^2$  is the variance of trend growth deviations from its historical rate of drift. Like the HP filter, the user of the PC filter can specify the relative weight placed on the two beliefs by constraining the ratio of the two variance terms to be equal to a specific value, given by  $k$  – as shown in Equation 10.

$$k = \sigma_1^2 / \sigma_2^2 \quad (10)$$

- 3.21 The parameter  $k$  is set to 625 in what follows – as a rough guide, this setting implies that shocks to the output gap are around five times as large as those to the level of potential output. The PC filter is estimated in state-space using the Kalman algorithm. To assist this algorithm in its search, it is helpful to set starting values for the unobserved states – I set these to be consistent with the output gap being closed at the start of the sample. In practice, this only affects estimates of the output gap very early in the real-time sample – both the real-time estimates at the end of the sample and the ex-post estimates across the whole sample are not sensitive to the choice of initial conditions.

<sup>8</sup> The PC filter is applied by Benes and N'Diaye (2004) and Laxton et al (1998), for example.

3.22 The state-space representation of the PC filter is given by Equations 11 to 14.

$$\text{Signal: } y_t = y_t^* + c_t \quad (11)$$

$$\text{State: } y_{t+1}^* = y_t^* + \text{drift}_t + \left( \frac{\varepsilon_{1,t}}{k^{1/2}} \right) \quad (12)$$

$$\text{State: } \text{drift}_t = \text{drift}_{t-1} \quad (13)$$

$$\text{State: } c_t = \varepsilon_{1,t} \quad (14)$$

3.23 As with the HP filter, the smoothing parameter,  $k$ , is important, but the PC filter is also consistent with a number of other implicit assumptions and judgements:

- the data-generating process governing the evolution of potential output is assumed to be a random walk with constant drift;
- from the above, the best guess of tomorrow's potential output growth is the historical rate of drift;<sup>9</sup>
- the output gap is an independent and identically distributed random variable, so the best guess of tomorrow's output gap, given today's output gap, is zero (no expected persistence); and
- shocks to demand are not correlated with shocks to supply.

3.24 The assumptions outlined above are similar in nature to those of the HP filter, with the main difference being that the HP filter is consistent with stochastic drift (shocks occur both to the level of potential output and its underlying growth rate) while the PC filter is consistent with constant drift (shocks occur only to the level of potential output, not its growth rate). In principle, any prior over the drift term can be incorporated, including structural breaks to the growth of potential GDP.<sup>10</sup> The remaining assumptions are subject to the same criticisms as described in the HP section.

3.25 With an appropriate choice of scaling parameter, the PC filter gives identical two-sided (ex-post) estimates of the output gap to those provided by the HP filter but it provides different real-time estimates (at the end of the sample). Both filters choose potential output such that their loss functions are minimised. The HP filter is based on a more flexible assumption for the dynamics of potential output than is the PC filter. So minimising the first part of its objective function (closing the output gap) is easier, because it is less costly to do so by adjusting its estimates of potential output growth.

<sup>9</sup> The one-sided filter calculates the historical average rate of drift up to the current time period and updates this as it moves forward. The two-sided filter makes an estimate of the drift term over the whole sample.

<sup>10</sup> Incorporating a structural break during the recent financial crisis does not affect the output gap estimate, rather it changes the path of supply shocks. So this judgement is more important for forecasting potential GDP than it is for estimating the output gap.



3.26 It is important to recognise that the assumptions underpinning different filters significantly affect their real-time properties. The HP filter is far more likely to signal a closed output gap at the end of the sample and be revised subsequently than is the PC filter – a feature thought by many to be undesirable. But, by placing more weight on past growth as a guide to future growth, the PC filter is slower to respond should there be structural breaks in the growth rate of potential output, and subsequently be revised for this reason – again, an undesirable property. Because of this, the real-time estimates of different filters might be more reliable at different times and care should be taken to consider other evidence.

Chart 3.5: PC output gap

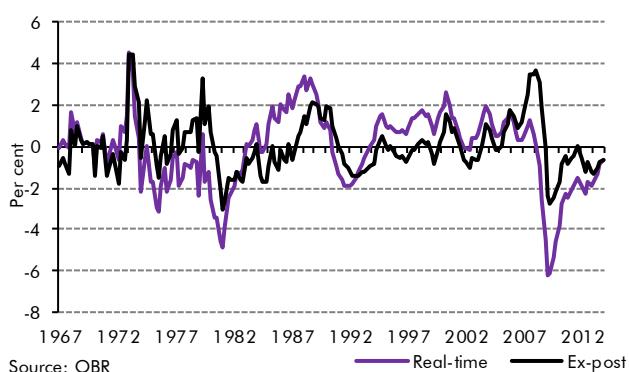
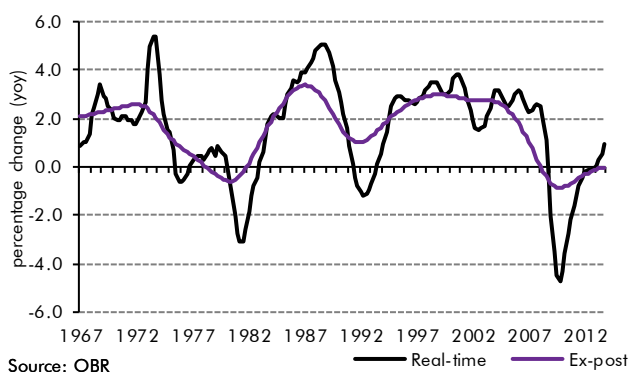


Chart 3.6: PC potential growth



3.27 Like the HP filter, the PC filter penalises volatility of trend growth so revisions to its output gap estimates, with the benefit of hindsight, imply more overheating in the economy before the recent financial crisis and slower trend growth in the years that preceded it. The key distinction between the real-time HP and PC filters is that the latter is more likely to interpret growth rates above the historical average as being unsustainable in real time than would the former. With the benefit of hindsight, the PC filter estimates are very close to those of the HP filter.

3.28 Starting the sample later has a small effect on an estimate of the output gap now – shifting the sample on five years alters the current output gap estimate by 0.1 percentage points, for example. Largely this is because the filter uses estimates of historical trend growth to inform its estimates, which is affected by the choice of sample period.

## Beveridge-Nelson (BN) decomposition

3.29 The trend-cycle decomposition of Beveridge & Nelson (1981) presents output as an autoregressive integrated moving average (ARIMA) process. They postulate that the permanent component of the series is equal to the long-run forecast of output, taking into account its mean rate of change – which identifies the trend and cycle components of output.

3.30 The BN decomposition depends upon a number of assumptions:

- output growth is stationary;

- the trend is equal to the long-run forecast of the series;
- both trend and cycle are affected only by a common shock; and
- the ARIMA specification is correct.

**3.31** The most important assumption is that all movements in the trend and cycle components of output are driven by a common (unidentified) shock. The shock to potential output is assumed to be negatively correlated with the cyclical shock – when the shock pushes potential output up, it pushes aggregate demand down. This is a fairly restrictive assumption but one possibility is that the shock could be accounted for by movements in productivity – an interpretation which could be consistent with a ‘real business cycle’ view of the world.<sup>11</sup> The results are also sensitive to the specification of the ARIMA model – Canova (1998) shows that the inclusion of more or fewer lags can greatly influence the resultant output gap estimates.

**3.32** I estimate the output gap using an ARIMA (2,1,0) specification and the estimates are presented in Chart 3.9. The combination of assumptions described above gives an output gap that is generally of smaller amplitude than its comparators, while the Beveridge-Nelson estimates of potential output are more volatile than actual output.

## Christiano-Fitzgerald (CF) filter

**3.33** The Christiano-Fitzgerald (CF) filter is a band-pass filter, formulated in the frequency domain. It works by filtering out data according to its frequency, decomposing a time series into trend, cycle and noise. In what follows, anything with a frequency below two years is considered noise, between 2 and 8 years is cycle and over 8 years is trend. This is a typical convention, but, like the HP filter, there is no strong evidence to bring to bear on the choice of cut-offs, so this choice is a judgement.

**3.34** The CF filter makes use of the entire sample to estimate the cycle and is subject to the same end-point problem as the other filters – the absence of future data makes it prone to revision. However, Nilsson & Gyomai (2011) find that the CF filter revisions tend to be a little smaller than those for the HP filter. The cost, though, is that it is less likely than the HP filter to pick up signals of turning points. They judge that the HP filter is more appropriate to the OECD’s short-term forecasting needs than is the CF filter. Furthermore, Estrella (2007) compares the performance of a range of univariate filters and finds that the HP filter performs best but, crucially, only in cases when its assumptions are consistent with the true process being examined. As with the BN decomposition, because the CF filter has no state-space representation, I refrain from presenting the real-time gap estimates, but the ex-post estimates are included in comparison Chart 3.9.

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<sup>11</sup> See Kydland & Prescott (1982) for a description of real business cycle theory.

## Comparison of univariate methods

Chart 3.7: Real-time output gaps

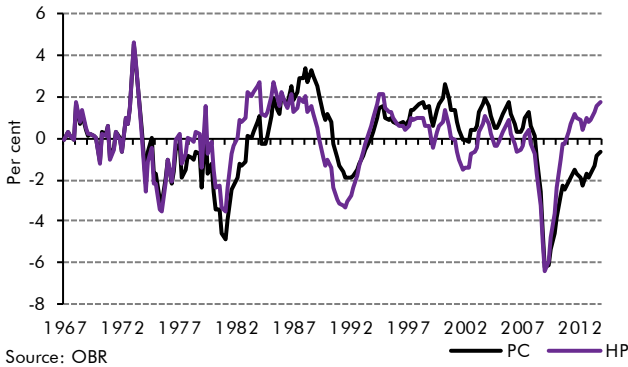
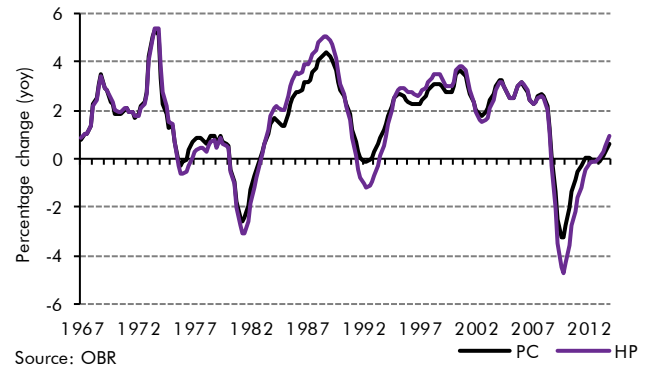


Chart 3.8: Real-time potential growth



**3.35** The two univariate filters estimated in real time are the HP and PC methods, illustrated in Charts 3.7 and 3.8. It is clear to see that potential growth is anchored more closely to the historical average using the PC filter than with the HP filter, though the broad pictures painted by both are similar. The resulting output gap series illustrate the specific point that the HP filter assumptions lead to bias at the very end of the sample. The very large negative output gap serves to bias down potential growth and so the HP filter persistently estimates a rapidly closing or even positive output gap at the end of the sample – most obvious during the present recovery period.

Chart 3.9: Ex-post output gaps

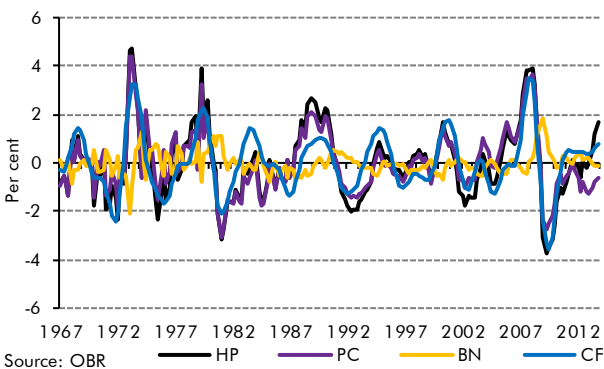
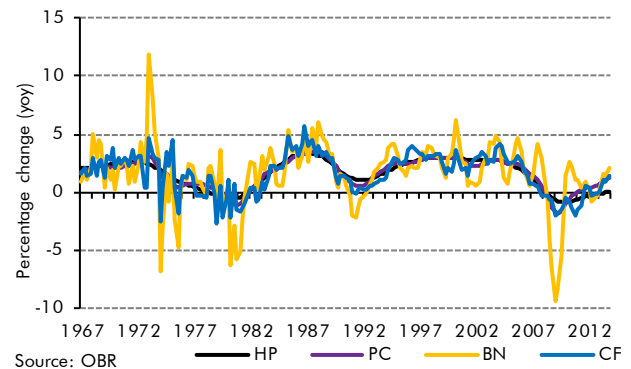


Chart 3.10: Ex-post potential growth



**3.36** The ex-post estimates of the output gap and potential growth include the BN decomposition and CF filter – presented in Charts 3.9 and 3.10. Most striking is how the different assumptions lead to such a wide range of estimates. Clearly, the BN decomposition is a different class of model altogether, consistent with significant potential output volatility (more so even than actual output). The PC and HP filters present similar estimates (except at the end of the sample, when the ex-post estimates converge with the real-time estimates). The CF filter is consistent with a smoother output gap series but more volatile potential output (since the volatility of actual output – the noise identified by the filter – must be assigned

somewhere), although a different set of cycle-length assumption would give a different set of estimates.

- 3.37 In summary, the results show that estimates of the output gap obtained using univariate filters are sensitive to the assumptions underpinning them. As Estrella (2007) notes, filters must be carefully selected for any particular application and no single method can accommodate all circumstances well.

## 4 Estimating the output gap: multivariate methods

- 4.1 This section covers a number of methods which make use of more than one variable. It starts by setting out a suite of models based on the multivariate PC filter, which is adapted from the multivariate HP filter presented in Laxton & Tetlow (1998). It then presents alternative methods including principal components and an aggregate composite of survey indicators, similar to those used in current OBR economic forecasts.<sup>12</sup>
- 4.2 Ultimately, the univariate filters set out in the previous section rely on judgement over the amplitude of the cycle and strong priors over the dynamics of potential output. To refine those judgements, we need more information. Often, though, taking on more information requires explicit assumptions over how it should influence estimates of the output gap, as is shown below.

### The multivariate PC filter

- 4.3 The standard multivariate filter, presented by Laxton & Tetlow, augments the objective function of the HP filter with the sum of squared residuals from another signal relationship. In what follows, I follow the same approach but use the PC filter instead, to avoid the specific type of end-point bias associated with the HP filter. The objective function of the filter (15), therefore, looks much like (9), but also includes the sum of squared residuals,  $\varepsilon_{2,t}$ , from a relationship that includes the output gap – that is, the filter also chooses the path of the output gap that most improves the fit of the hypothesised relationship.

$$\sum_{t=1}^T \left( \frac{1}{\sigma_1^2} (c_t)^2 + \frac{1}{\sigma_2^2} (\Delta y_{t+1}^* - \text{drift}_t)^2 + \frac{1}{\sigma_3^2} (\varepsilon_{3,t})^2 \right) \quad (15)$$

- 4.4 And so, the multivariate PC filter is based on three beliefs, the first two of which are common to the standard PC filter:

- output does not deviate too far from its trend level (cycles are not too big);
- the growth rate of potential output does not differ too much from its historical average rate of drift; and
- other indicators, for example unemployment, tell us something about the cyclical position of the economy.

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<sup>12</sup> More details are set out in OBR (2011) and Pybus (2011).

4.5 While taking on information from other sources is likely to ameliorate, to some extent, the end-point problem, judgements are still required. Instead of one smoothing parameter to choose, there are now two. These are used to decide, first, the variability of potential output and, second, how much weight to place on the structural relationship of choice. In practice, this allows us to select a smoothness of potential output that is consistent with the time horizon appropriate for use by a fiscal authority. But it also allows us to make use of other information, which might be more relevant to a medium-term concept of potential output (such as inflation). The two parameters are given by:

$$k = \frac{\sigma_1^2}{\sigma_2^2} \quad (16)$$

$$\text{and} \quad \varphi = \frac{\sigma_1^2}{\sigma_3^2} \quad (17)$$

4.6 While the scaling parameter in Equation 16 sets the variability of potential output relative to the sum of squared residuals of the output gap equation, the weight placed on other information, the specification and the fit of the structural equation of choice will also influence the overall cyclicity of the estimated output gap series. So, in what follows, the cyclicity of the resultant series is inspected relative to the baseline case and any major differences reported. The variability of potential output is tied down judgementally by setting  $k$ . The weight placed on information from the structural relationship is determined by the size of  $\varphi$  relative to  $k$ . As a baseline case,  $\varphi$  and  $k$  are set equal to one another. Alternatively, judgement about how much weight to place on other information can be applied by adjusting the second parameter, given by Equation 17.

4.7 In what follows, I augment the PC filter by one of three possible relationships:<sup>13</sup>

- a Phillips curve;
- a version of Okun's law; and
- a capacity utilisation equation.

## Philips curve-augmented PC filter

4.8 Inflation could contain useful information about slack in both the labour and product markets. First, tightness in the labour market could see the pass-through of higher wage demands to prices. Second, excess demand in the product market could affect firms' mark-up decisions. To make use of this information, the model presented below includes a reduced-form version of a structural relationship between inflation and spare capacity in the economy, known as the New Keynesian Phillips curve – similar to that presented in Gali & Monacelli (2005) and presented in Equation 18:

$$\pi_t = \beta_1 \pi_{t+1}^e + (1 - \beta_1) \pi_{t-1} + \beta_2 c_{t-1} + \varepsilon_{3,t}. \quad (18)$$

---

<sup>13</sup> In some ways, this method is similar to the MV method presented in Benes *et al* (2010), although I choose to calibrate the model parameters and place weights on the information, rather than use Bayesian methods to restrict the parameter space and specify priors over shocks to the measurement equations.

- 4.9  $\pi_t$  is the deviation of inflation from its steady state rate,  $\pi_{t+1}^e$  is the expected deviation one period ahead and  $\pi_{t-1}$  is lagged inflation. This formulation introduces persistence to the inflation process, such that it better matches the observed data. The neutrality of money is preserved by constraining the sum of the coefficients on future and lagged inflation to unity. The influence of the cycle,  $c_{t-1}$ , is captured by the coefficient  $\beta_2$ . In this model, inflation expectations are anchored to the steady state inflation rate, so the expected deviation of inflation from steady-state is zero (i.e.  $\pi_{t+1}^e = 0$  in (18)).
- 4.10 The chosen measure of inflation is CPI inflation, adjusted for the estimated effects of VAT measures (which are not directly related to the cycle) and the influence of food and oil costs, which are volatile and exogenous. The model parameters are calibrated in line with Murray (2012).
- 4.11 The steady state inflation rate is derived by applying the PC filter to the inflation series and the data-generating process is assumed to be a random walk without drift (the drift term is assumed to be zero in the objective function of the filter). Real-time estimates of the output gap use real-time estimates of steady-state inflation and likewise ex-post estimates are based on the ex-post steady-state estimate. The state-space model is given by Equations 19 to 23.

$$\text{Signal: } y_t = y_t^* + c_t \tag{19}$$

$$\text{Signal: } \pi_t = (1 - \beta_1)\pi_{t-1} + \beta_2 c_{t-1} + \left( \frac{\varepsilon_{1,t}}{\varphi_\pi^{1/2}} \right) \tag{20}$$

$$\text{State: } y_{t+1}^* = y_t^* + \text{drift}_t + \left( \frac{\varepsilon_{1,t}}{k^{1/2}} \right) \tag{21}$$

$$\text{State: } \text{drift}_t = \text{drift}_{t-1} \tag{22}$$

$$\text{State: } c_t = \varepsilon_{1,t} \tag{23}$$

Chart 4.1: Real-time output gaps

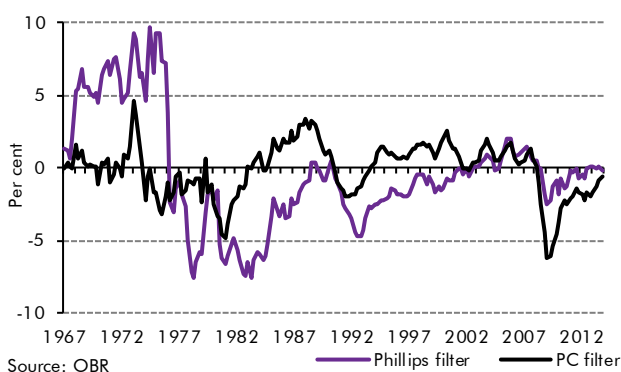
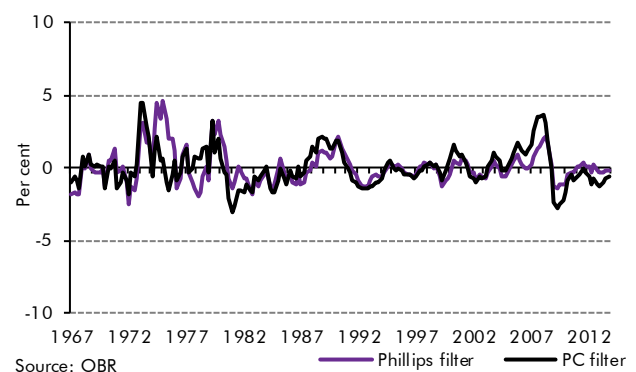


Chart 4.2: Ex-post output gaps



4.12 The real-time and ex-post output gap series estimated using the Phillips curve to inform the judgement are compared with those of the standard PC filter in Charts 4.1 and 4.2. The real-time estimates illustrate the uncertainty associated with real-time estimates of the trend inflation rate and, to some extent, the challenge of removing exogenous shocks from the series – particularly oil price shocks in the 1970s. The ex-post estimates look more sensible, since the trend inflation rate is known with more certainty over the difficult periods.

4.13 When inflation is taken into account, the late-1980s boom looks a bit smaller than the PC filter implies ex-post. The low and stable inflation environment from the late 1990s serves to anchor the Phillips estimates of the output gap close to zero, but the inclusion of inflation tells us little about the output gap over that period on an ex-post basis. Overall, the ex-post estimates from the Phillips curve are relatively close to those of the basic PC filter, but the weakness of inflation before the last crisis and absence of strong disinflationary pressure afterwards tends to reduce both the estimated size of the boom and bust.

Chart 4.3: Real-time potential growth

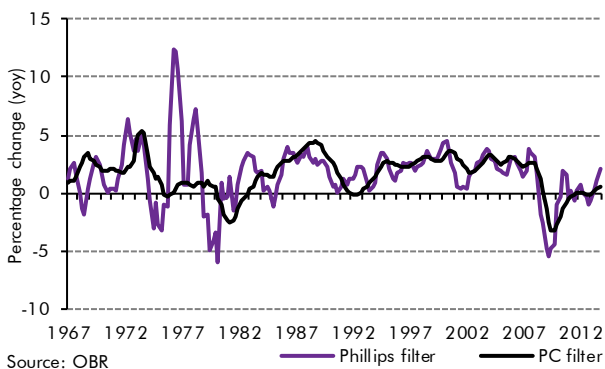
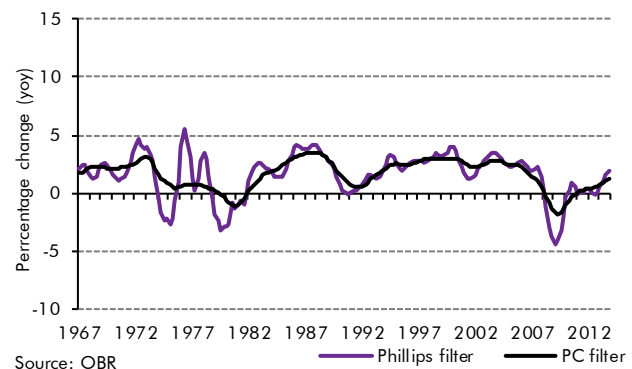


Chart 4.4: Ex-post potential growth



4.14 The additional information from inflation tends to increase the volatility of trend growth, relative to the estimates provided by the basic filter. This is because less weight is placed on minimising deviations from the historical rate of growth, in order to take on information from inflation. In extremis, a weight could be chosen that would make the output gap look exactly like the inflation deviation from steady-state and the potential growth rates would be much more volatile to reflect this.

4.15 The real-time estimates provided by this method are probably only reliable towards the end of the sample, when the monetary policy regime became more settled. The ex-post estimates are useful insofar as they help to incorporate information from inflation into estimates of the output gap while broadly preserving the features of the cycle. But incorporating such information requires a number of assumptions and implicit judgements, including but not limited to:

- the hypothesised Phillips relation being correctly specified;
- the calibrated coefficients of the Phillips curve being correct;
- stability of the relationship over time;



- the filter-based estimate of steady-state inflation being accurate; and
- the prior weight placed on its information content being appropriate.

4.16 These assumptions are highly uncertain – there is ongoing contention over how the Phillips curve should be specified – see Rudd and Whelan (2007) for example. The coefficients are poorly identified and circularity is introduced because they are typically estimated using output gap estimates. The relationship may not be stable over time – see Iakova (2007) for evidence on the flattening of the Phillips curve. The steady-state is estimated using a relatively stiff filter but there have been significant changes to the monetary policy framework over the sample period. And finally, given the uncertainties, it is unclear how much weight should be placed on the Phillips curve.

4.17 Neither changing the starting values nor shifting the sample on five years make a material difference to the estimates of the gap at the end of the sample period. Partly, the small difference reflects the fact that the key parameters are calibrated rather than estimated so model uncertainty does not have an influence when the sample is changed. The average output gap revision is around 2.6 percentage points, which is very large, but this largely reflects uncertainty over the trend rate of inflation. By selecting a sample over which the monetary policy framework was stable (1995 onwards, for example), this falls to 0.8 percentage points.

## Okun’s law augmented PC filter

4.18 To take account of slack in the labour market, we can make use of another relationship – that between cyclical unemployment and the output gap. This simple relationship is set out in Bailey & Okun (1965) and a version of it is presented in Equation (24), where  $uc_t$  is the cyclical deviation of unemployment from its natural rate and  $\varepsilon_{4,t}$  is an i.i.d shock:

$$uc_t = \beta_3 c_t + \varepsilon_{4,t}. \quad (24)$$

4.19 To ensure consistency across the various measures presented that use the unemployment gap as an indicator, the real-time and ex-post estimates of the structural rate (the NAWRU) are produced using a common methodology, described in the Annex. I use these measures of the NAWRU to inform the estimate of the output gap. As with the Phillips model, the Okun relationship is added to the state-space model of the PC filter:

$$\text{Signal: } y_t = y_t^* + c_t \quad (25)$$

$$\text{Signal: } uc_t = \beta_3 c_t + \left( \frac{\varepsilon_{1,t}}{\varphi_u^{1/2}} \right) \quad (26)$$

$$\text{State: } y_{t+1}^* = y_t^* + \text{drift}_t + \left( \frac{\varepsilon_{1,t}}{k^{1/2}} \right) \quad (27)$$

$$\text{State: } drift_t = drift_{t-1} \tag{28}$$

$$\text{State: } c_t = \varepsilon_{1,t} \tag{29}$$

Chart 4.5: Real-time output gaps

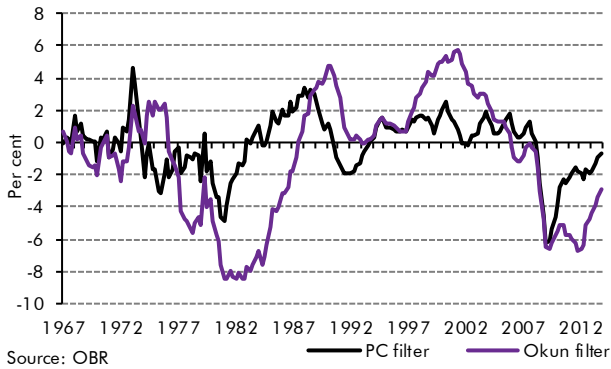
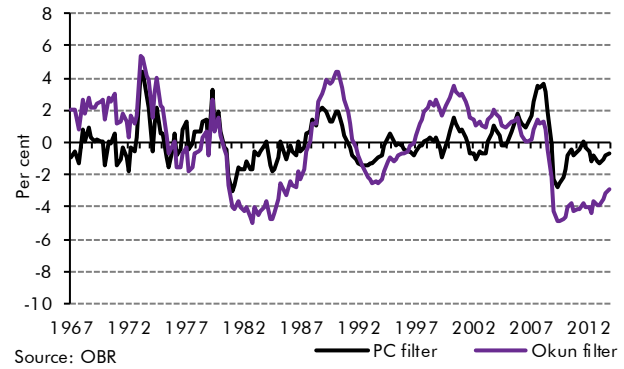


Chart 4.6: Ex-post output gaps



4.20 Charts 4.5 and 4.6 compare output gap estimates from the Okun-augmented PC filter with those from the basic PC filter. The Okun filter sees more slack in the late 1970s and early 1980s than does the PC filter, which accords with elevated unemployment over that period and ultimately, the ex-post estimates from the PC filter are revised closer to the Okun estimates. The late 1980s boom appears to be more pronounced in real-time, using the Okun filter and, again, the PC filter shows a sharper boom at the end of the 1980s ex-post.

4.21 Making use of the Okun relationship, the elevated rate of unemployment widens the estimated output gap in the period since the recent recession and makes the boom that preceded it look small. Overall, the results suggest that it is important to recognise such an obvious source of spare capacity in the economy when forming a view of the output gap.

Chart 4.7: Real-time potential growth

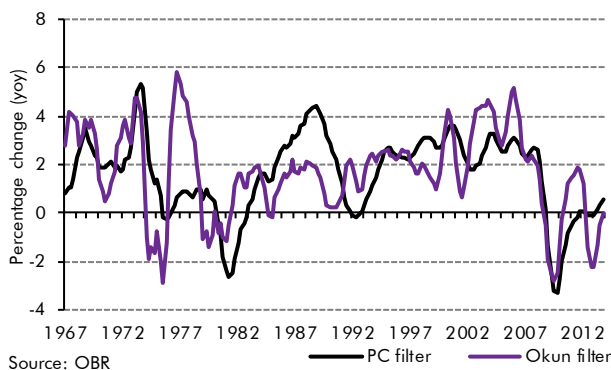
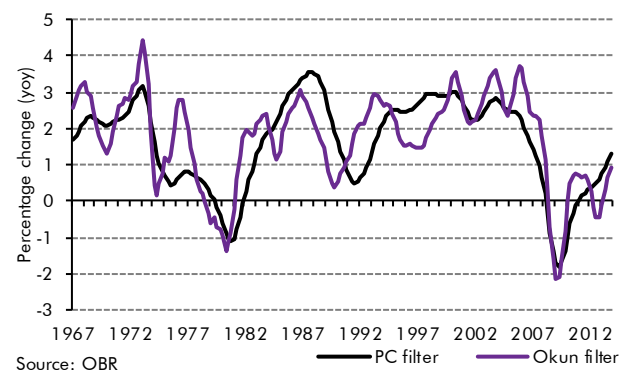


Chart 4.8: Ex-post potential growth



4.22 But, as with the Phillips relation, it is important to remember that the estimates depend heavily on a number of assumptions:

- the hypothesised Okun relation being correctly specified;
- the estimated coefficients of the Okun relation being correct;

- stability of the relationship over time; and
- the filter-based estimate of the NAWRU being accurate.

4.23 Again, these assumptions are highly uncertain. The Okun relation has various specifications and, while the coefficient of -0.5 is estimated over the whole sample period, there is evidence to suggest that it may vary – the recessions of the 1980s and 1990s were associated with far larger increases in unemployment than was the latest recession. If the Okun coefficient were now lower, then the estimated output gap would be biased in a way that made it appear larger.

4.24 Moving the sample on five years has a small effect on the estimates of the current output gap (a bit less than 0.1 percentage points), while the estimates of the output gap are revised, on average, by around 1.3 percentage points – a bit less than the average magnitude of the output gap (around 1.7 per cent).

## Capacity utilisation-augmented PC filter

4.25 The third augmentation relies on a posited, non-structural relationship between capacity utilisation indicators and the output gap. This is intended to capture slack within firms.

$$\text{capc}_t = \beta_4 c_t + \varepsilon_{5,t}. \quad (30)$$

4.26 This estimate of the output gap is estimated using the system of Equations 31 to 35:

$$\text{Signal: } y_t = y_t^* + c_t \quad (31)$$

$$\text{Signal: } \text{capc}_t = \beta_4 c_t + \left( \frac{\varepsilon_{1,t}}{\varphi_{cap}^{1/2}} \right) \quad (32)$$

$$\text{State: } y_{t+1}^* = y_t^* + \text{drift}_t + \left( \frac{\varepsilon_{1,t}}{k^{1/2}} \right) \quad (33)$$

$$\text{State: } \text{drift}_t = \text{drift}_{t-1} \quad (34)$$

$$\text{State: } c_t = \varepsilon_{1,t} \quad (35)$$

4.27 The capacity utilisation data are sourced from the Confederation of British Industry (CBI) and pertain to manufacturers only – services firms, which account for the bulk of activity, have not been surveyed over the long time series available for the manufacturing sector.<sup>14</sup> For simplicity, the weight placed on the equation in explaining the output gap is set equal to the smoothing parameter, as with the Phillips and Okun filters.

<sup>14</sup> As with the other methods, the capacity utilisation data have been pre-filtered to extract trend and cycle – both in real time and ex post.

Chart 4.9: Real-time output gaps

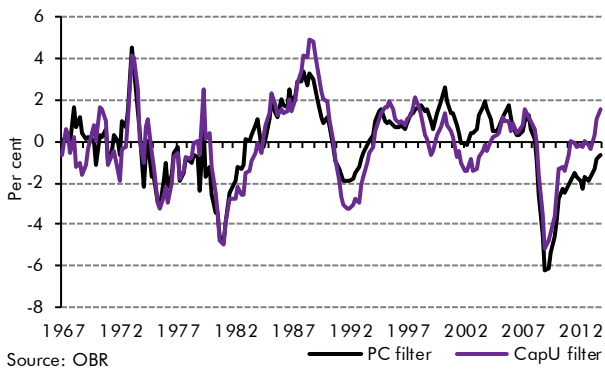


Chart 4.10: Ex-post output gaps

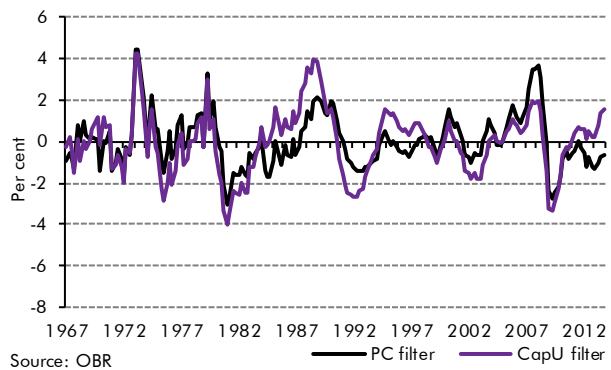


Chart 4.11: Real-time potential growth

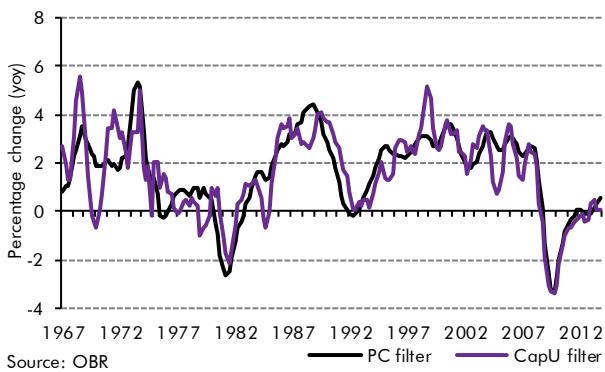
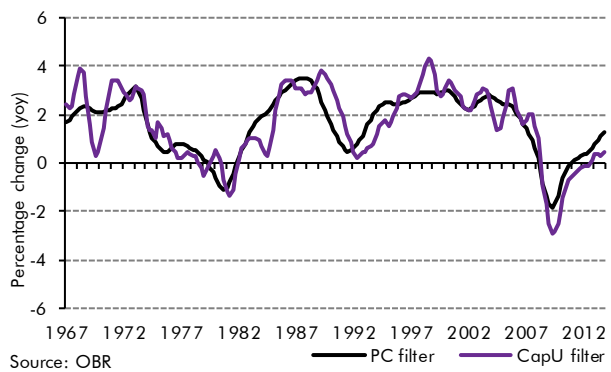


Chart 4.12: Ex-post potential growth



4.28 The inclusion of capacity utilisation serves to make the 1980s boom appear larger than it does in the basic PC filter. And, because capacity utilisation was close to usual levels before the latest recession, it makes the positive output gap smaller. It also points to the economy operating significantly above trend from 2010 onwards. Again, this model depends on a number of non-trivial judgements and assumptions:

- the hypothesised capacity utilisation relation being correctly specified;
- the freely-estimated coefficient being correct;
- stability of the relationship over time; and
- the steady-state survey balance being correctly estimated.

4.29 Again, these assumptions are highly uncertain – there is no structural link between survey balances and the output gap, rather it should be treated as an indicator. It is also difficult to know exactly how respondents interpret the survey question. When asked, they may be thinking of a very short-term notion of spare capacity and ignore mothballed capacity that could be brought back online in the medium term. This may lead to biases in the short term that later unwind, but these would not be captured in the methodology described above. Furthermore, the survey captures the number of firms operating below/above capacity,

rather than the extent to which firms are operating above/below capacity – the latter is what is relevant to output gap estimation.

- 4.30 Shifting the sample start date forwards by five years has a small effect on the size of the output gap estimate for the final quarter of 2013 – around 0.1 percentage points. The revisions are small because the capacity utilisation data are not revised and abstract from uncertainty over the steady state values.

## Multivariate filter model

- 4.31 The final incarnation of the PC filter in this paper takes information from all three of the output gap relationships described above and forms a multivariate filter that is estimated using the system of Equations 36 to 42:

$$\text{Signal: } y_t = y_t^* + c_t \quad (36)$$

$$\text{Signal: } \pi_t = (1 - \beta_1)\pi_{t-1} + \beta_2 c_{t-1} + \left( \frac{\varepsilon_{1,t}}{\varphi_\pi^{1/2}} \right) \quad (37)$$

$$\text{Signal: } uc_t = \beta_3 c_t + \left( \frac{\varepsilon_{1,t}}{\varphi_u^{1/2}} \right) \quad (38)$$

$$\text{Signal: } capc_t = \beta_4 c_t + \left( \frac{\varepsilon_{1,t}}{\varphi_{cap}^{1/2}} \right) \quad (39)$$

$$\text{State: } y_{t+1}^* = y_t^* + drift_t + \left( \frac{\varepsilon_{1,t}}{k^{1/2}} \right) \quad (40)$$

$$\text{State: } drift_t = drift_{t-1} + \varepsilon_{1,t} \quad (41)$$

$$\text{State: } c_t = \varepsilon_{1,t} \quad (42)$$

- 4.32 The parameter governing the volatility of potential output,  $k$ , is set to 625 but there are now three other parameters to select based on the weight placed on them in explaining the output gap. The weights chosen here are all equal – setting the parameters  $\varphi_\pi$ ,  $\varphi_{cap}$  and  $\varphi_u$  to 625. Charts 4.13 and 4.14 illustrate the effect of including additional information from hypothesised relationships between the output gap and inflation, unemployment and capacity utilisation. The weight placed on additional information in forming a view on the output gap serves to reduce the end-point problem of the filter, insofar as the MV model places less weight on GDP data from the future and so is less susceptible to revision when it comes to bear.

Chart 4.13: Real-time output gaps

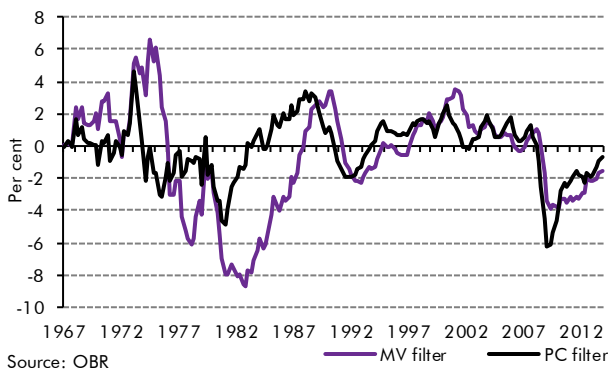
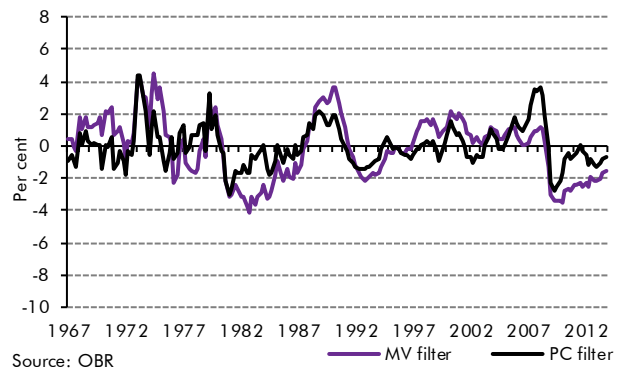


Chart 4.14: Ex-post output gaps



4.33 The most notable difference between the MV and PC estimates is that the size of the pre-crisis boom looks substantially smaller, because much of the data used to augment the filter were consistent with output being close to trend. The information does suggest, however, that output was further above trend in the late 1980s and the late 1990s than the basic filter would suggest.

Chart 4.15: Real-time potential growth

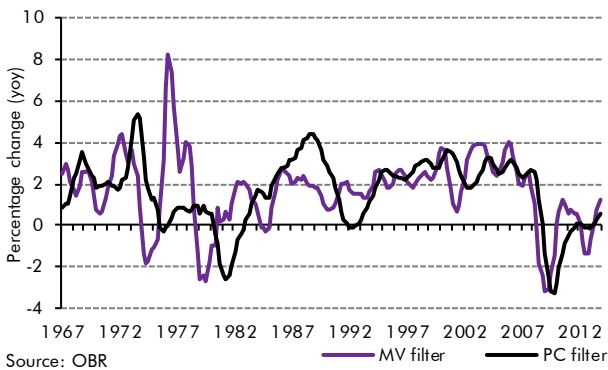
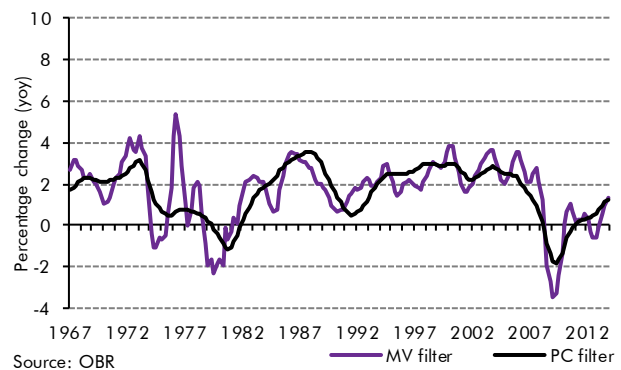


Chart 4.16: Ex-post potential growth



4.34 Potential growth is somewhat more volatile using the MV filter, because less weight is placed on preserving its smoothness once other data are taken into account. In principle, a higher weight could be placed on minimising the deviation of potential growth from trend and this would serve to make the MV filter estimate closer to the PC estimate. Ultimately it is a matter of judgement as to how volatile potential output should be, since it cannot be observed – see Box 6.1.

4.35 While the MV filter makes use of more information, the judgements and assumptions required to make the most of it begin to add up, at the cost of reduced transparency. As well as the weights placed on the value of each relationship, their specification and underpinning assumptions all affect the resulting output gap series. So, in some sense, the weight placed on them is arbitrary. The myriad uncertainties and assumptions suggest that a method such as this, and variants of it, should be considered as part of a suite of indicators rather than providing a single point estimate.

## Principal components analysis

- 4.36 Principal components analysis (PCA) is a statistical technique that attempts to draw a common signal from a range of de-meaned, standardised cyclical indicators – which can consist of survey measures of capacity utilisation, recruitment difficulties or inflation, for example. The estimation process involves assigning weights to each of the data series such that the resultant output gap series explains as much of the variability of the dataset as possible. For further information on the construction of the PCA see Pybus (2011).
- 4.37 Once a principal component has been estimated, its mean and standard deviation (a scaling parameter) must be chosen. These choices are the main judgements upon which PCA estimates of the output gap depend. One method would be to set the standard deviation such that the amplitude of the business cycle accords with some prior judgement. Another would be to try to replicate the standard deviation of another output gap series.
- 4.38 Chart 4.17 illustrates the path of the output gap implied by the PCA, where the scaling parameter and mean have been set in line with the standard deviation and mean of the PC-filtered series (to maintain comparability). The PCA method estimates the output gap directly from the cyclical indicators, so it makes no effort to smooth trend growth – the additional volatility in trend growth relative to the PC filter is presented in Chart 4.18.

Chart 4.17: Ex-post output gaps

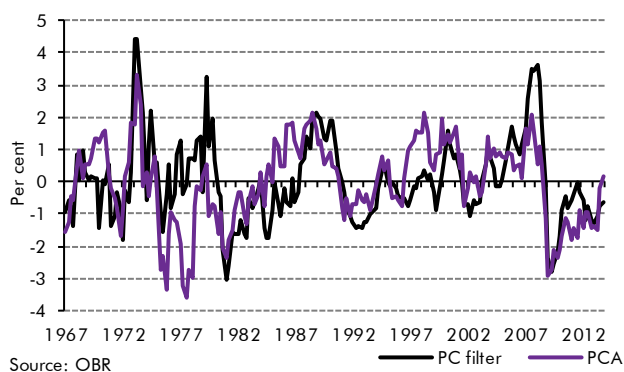
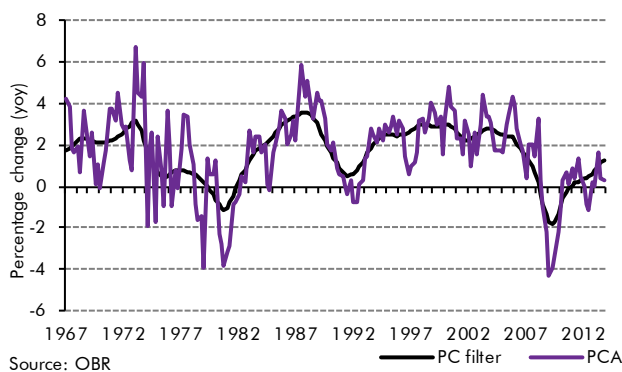


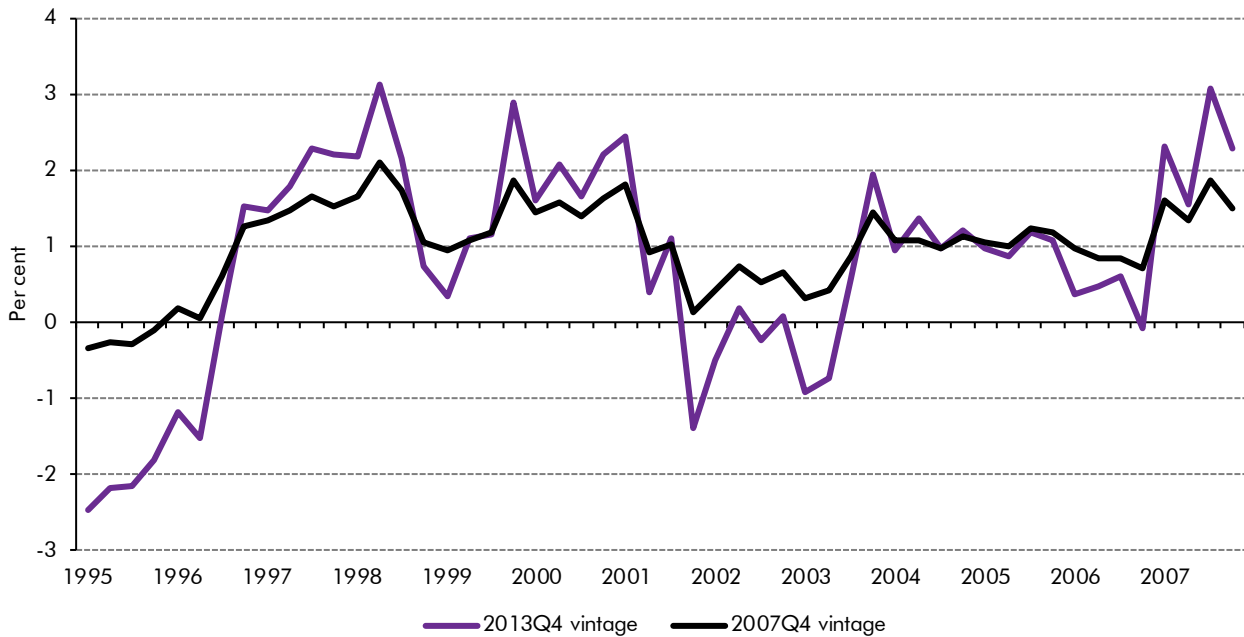
Chart 4.18: Ex-post trend growth



- 4.39 The PCA measures the output gap directly using data that are typically not revised, which means that the principal component it produces in real time is unlikely to differ very much from that which it estimates on an ex-post basis. However, the scaling parameter and mean might well change with the benefit of hindsight as these judgements are updated. It is difficult to generate a real-time series of estimates using the PCA method, but to get an indication of the scope for the PCA to be revised I consider two vintages of its estimates.
- 4.40 Chart 4.19 shows the PCA estimate that would be generated in real-time using data available up to and including the final quarter of 2007, with the standard deviation and mean set to those from the PC-filter estimate of the output gap up to that point. It also shows the PCA estimate using data up to the final quarter of 2013, with the standard deviation and mean set to those of the PC filter up to the same point. It is clear from the chart that the size of the pre-crisis boom now appears significantly larger than it did at the

time, for example. So the PCA is not very sensitive to data revisions but is sensitive to changes in judgement over the scaling parameter and mean, with an average revision of around 1.2 percentage points between the two vintages shown here.

Chart 4.19: Vintages of PCA output gaps



Source: OBR

## Aggregate composite

4.41 The aggregate composite (AC) measure of the output gap uses similar data to that of the PCA, described above. But, rather than estimating the weight to be placed on each series based on the correlation of the dataset, these are set explicitly by the user. I adopt the same methodology as Pybus (2011), which de-means and standardises each survey indicator. The aggregated composite is a weighted average of survey indicators of capacity utilisation and recruitment difficulties – where the weights are based on factor income and sector shares.

4.42 Like the PCA, the AC must be adjusted and scaled either subjectively or to match the mean and standard deviation properties of another output gap series. To construct the estimates presented in Charts 4.20 and 4.21, I set the mean and standard deviation so that it is consistent with the PC filter estimates. The series is also spliced together in the middle, since survey data for the services sector is unavailable prior to 1995 and the estimates are calculated using only manufacturing data prior to that point.



Chart 4.20: Ex-post output gaps

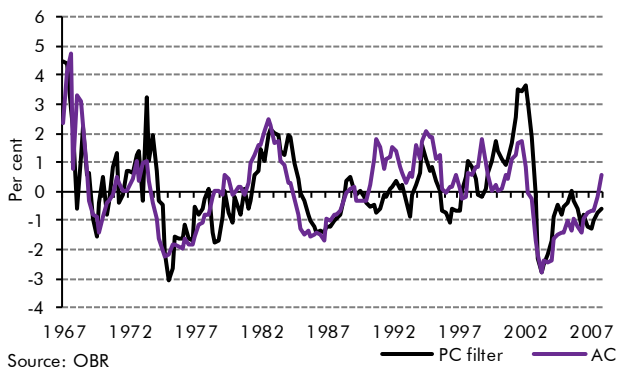
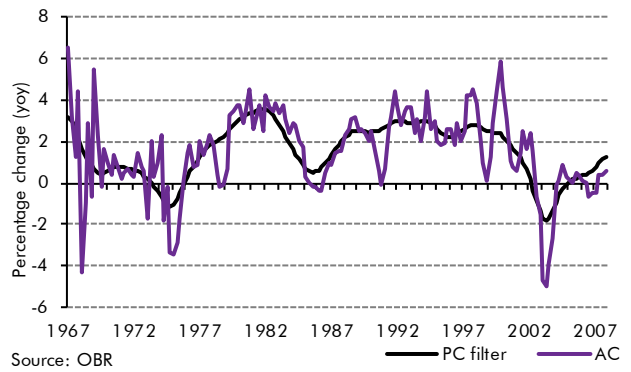


Chart 4.21: Ex-post trend growth



4.43 Overall, the results are fairly similar to the PCA measure of the output gap and potential output growth. Like the PCA, the AC does not place any explicit weight on smoothing potential output, but the amplitude of the cycle is constrained to be the same as the PC filter estimates. The revisions properties are likely to be very similar to the PCA (presented in Chart 4.19).

### Box 4.1: Output gap measurement – the role of credit

Before the 2008 financial crisis, there was little evidence of overheating in the UK economy – wage growth, inflation and unemployment were all at rates consistent with historical averages. Yet, most analysts and commentators now agree that there was something unsustainable about the strong growth in the years before the crisis hit – most estimates of the output gap imply a significant and permanent hit to the level of potential productivity.

The period before the crisis saw significant growth of credit. To the extent that the financial cycle and the business cycle are correlated, financial variables could be used to inform estimates of the output gap. Indeed, output gaps augmented in this way may better-explain the cyclical behaviour of tax receipts, since some are related to asset prices and financial transactions. However, there are a number of practical problems in capturing the influence of credit.

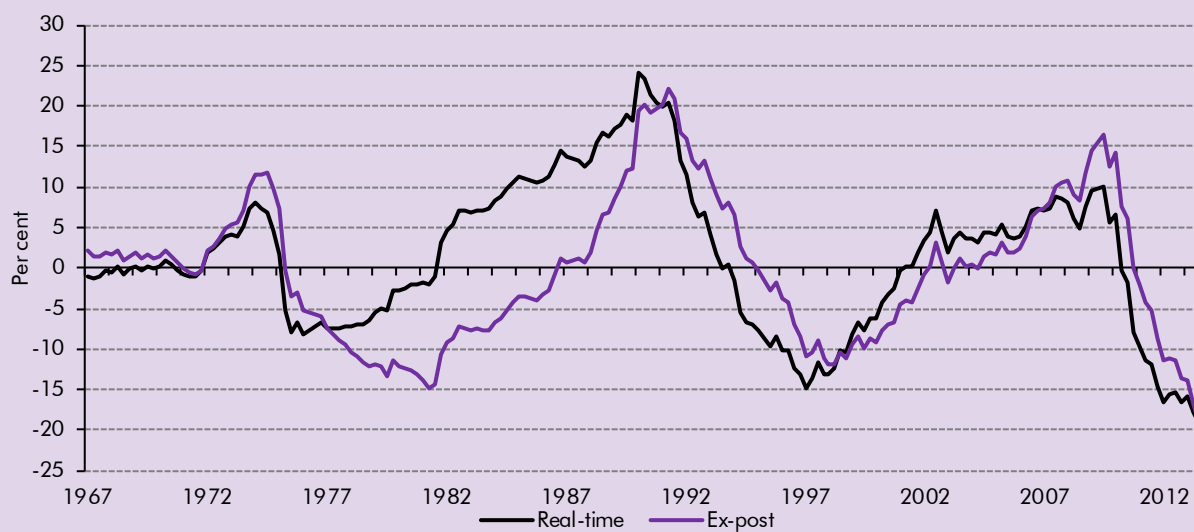
First, credit can rise as a share of GDP for structural reasons – deregulation in the 1980s increased mortgage availability while falling real interest rates in the 1990s made mortgage payments more affordable, each boosting owner-occupation rates and so net mortgage lending. For this reason, the Basel Committee on Banking Supervision (BCBS) excludes some forms of lending in its preferred measure. However, even excluding loans secured on dwellings, loans for direct investment and derivatives, the liabilities of households, corporations and not for profit entities rose from about 130 per cent of GDP in 2000 to around 170 per cent of GDP in 2007, before falling back during the recession.

Second, there is no direct relationship between credit and GDP (it is not a factor of production), and there is no structural relationship between the credit gap and the output gap. Instead it is treated as an indicator and how much emphasis to place on it requires judgement.

The BCBS-recommended credit gap measure, which the Bank of England publishes and the FPC uses to inform its policy decisions, is replicated in Chart A. It is a real-time series produced using the HP filter with the smoothing parameter set to 400,000 – for further details see Bank of England (2014b) and BCBS (2010). The series indicates a significant financial boom in the late 1980s and a smaller one before the recent financial crisis. I also present the ex-post series, which shares the same broad features as the real-time series but indicates that the 1980s credit boom starting somewhat later. It is also consistent with a bigger credit cycle in 2007/08 than it would have suggested using data available to policy makers at the time.

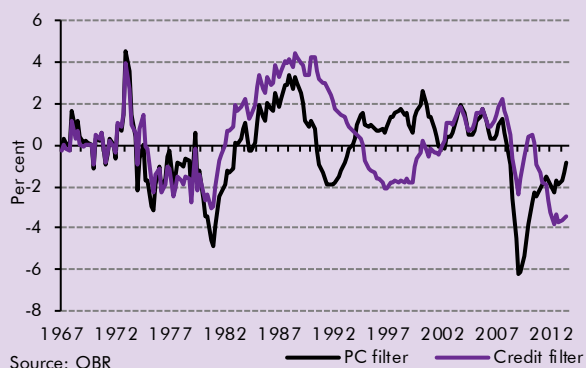
More broadly, this version of the credit gap does not tally especially well with conventional wisdom over the cyclical position of the economy. Because balance sheet movements are a relatively slow process, the credit gap is very persistent and recoveries can start long before leverage begins to rise. So a credit gap may help to inform judgements about the sustainability of growth and risk in the financial system but might be less useful as a real-time indicator of slack in the economy and, therefore, the scope for growth as it is taken up. This persistence is evident in the output gap estimates produced using an augmented PC filter (Chart B), in real time and the revised, ex-post estimates (Chart C).

Chart A: BCBS Credit Gap



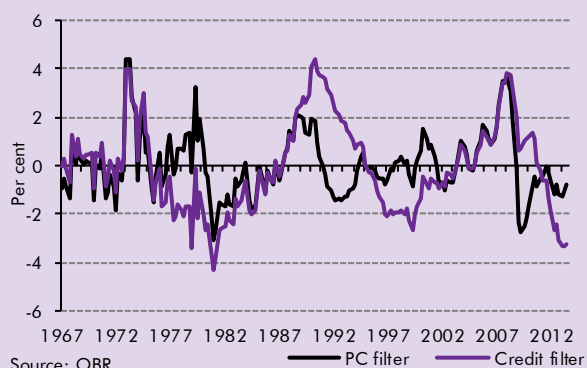
Source: OBR

Chart B: Real time output gaps



Source: OBR

Chart C: Ex-post output gaps



Source: OBR

An alternative method of taking on board information from the credit gap, when estimating the output gap, is to add it as a relevant indicator to a principal components analysis. However, PCA estimates are not drastically affected by the inclusion of the credit gap – largely this is because its low correlation with other variables means it is assigned a low weight when a common signal is extracted. There are still further ways in which to incorporate financial information into estimates of the output gap – see Borio *et al* (2013), for example – but each must grapple with the differing frequencies of the financial and output cycles.

## 5 Estimating the output gap: production function approach

5.1 A production function is simply an equation that relates inputs to the production process to outputs. That is to say, the level of potential output is a function of labour supply, the capital stock and the maximum efficiency with which they can be combined (total factor productivity (TFP)). To estimate the output gap, the actual level of output is compared with this potential level.

5.2 There are two key judgements associated with production function estimates of the output gap. The first is the choice of production technology – an economic theory that relates the inputs to the outputs. How might output be expected to change with the addition of another unit of capital or labour, for example? The second is the choice of method for estimating the potential levels of the factor inputs and total factor productivity, which are to be aggregated using the specified production technology.

5.3 The production technology can take many forms, but, in what follows, I follow the baseline Cobb-Douglas approach, Douglas (1948), and use standard assumptions. I assume that the production process is characterised by a function of the form:

$$y = A(L^\alpha K^{1-\alpha}). \tag{43}$$

5.4 A is TFP, L is labour input, K is the capital stock,  $\alpha$  and  $(1 - \alpha)$  are the output elasticities of labour and capital, respectively. This particular function is consistent with a number of assumptions:

- constant returns to scale;
- the marginal productivity of each factor being proportionate to its average productivity;
- technology being Hicks neutral – technological improvements increase the returns to labour and capital in equal measure; and
- under the additional assumption of perfect competition in the product market, factors are paid their marginal products, the steady-state labour share of income is, therefore, stable and can be used to calibrate the elasticity of factor inputs with respect to output.

5.5 The functional form of the Cobb-Douglas production function is largely one of convenience, since it is easy to work with. An alternative would be the, more general, Constant Elasticity of Substitution (CES) production function, which could be consistent with a trending labour

## Estimating the output gap: production function approach

share of income, for example.<sup>15</sup> However, as Miller (2008) shows, there is little evidence that supports the use of one over the other when the objective is to forecast GDP or factor shares, provided the labour share of income is stable. Absent cyclical fluctuations, the UK labour share of income appears broadly stable over the past few decades, so I stick with the more tractable Cobb-Douglas specification.

5.6 Now that a production technology has been selected, it is necessary to estimate the trend series for labour, capital and TFP. Almost all practitioners of this approach do this by using filters of some sort, so it stands to reason that the estimates it produces share many similarities with those obtained via the methods described earlier in this paper (rather than constituting an estimation technique in its own right).<sup>16</sup> It does, however, enable the user to interrogate the estimates with economic theory, by decomposing the output gap into contributions from TFP and the factors of production.

5.7 In what follows, labour supply is defined as potential hours worked and comprises potential average hours worked and potential employment. Potential employment, itself, consists of potential participation in the labour market as a share of potential population less the structural rate of unemployment – the NAWRU.

5.8 To estimate the trend series used to construct the production function, I:

- assume the population gap is zero at all times by setting potential equal to actual;
- apply a stiff PC filter to the activity rate to identify potential participation;
- take the NAWRU estimates used by the MV and Okun-augmented PC filters – estimated using the PC filter and a wage equation;
- apply a stiff PC filter with drift to actual average hours to obtain potential average hours;
- assume the potential capital input is equivalent to utilising the existing capital stock; and
- apply a PC filter with drift to the level of actual TFP to obtain potential TFP.

5.9 Clearly, the assumptions underpinning the production function estimates of potential output are numerous and susceptible to exactly the same sorts of issues identified in earlier filter applications. And any number of filter or specification choices could be made that could be expected to yield different estimates. The output gap is estimated by comparing the level of potential output with the actual value. Furthermore, the actual values of the factor inputs and TFP can be compared with their potential values, taking the production technology into

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<sup>15</sup> Barnes *et al* (2008) finds evidence using firm-level data that would support the use of a CES production function.

<sup>16</sup> The European Commission's approach is set out in D'Auria *et al* (2010), for example.

account, to give contributions to the output gap from each – the principal purpose of the exercise.

Chart 5.1: PF output gaps

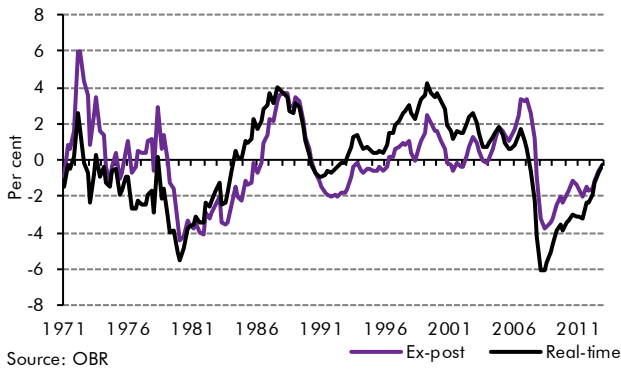
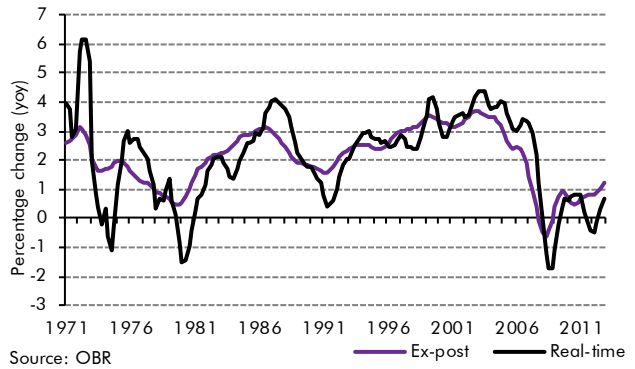


Chart 5.2: PF potential growth



5.10 Charts 5.1 and 5.2 illustrate the output gap and potential output growth paths produced by the production function approach. In many ways, they are unremarkable since they look similar to estimates produced by some of the methods described above. But the advantage of using a production function estimate is that we can decompose the output gap into contributions from the factor inputs and TFP.

5.11 Charts 5.3 and 5.4 illustrate two approaches to decomposing the output gap into contributions. The first is perhaps the most intuitive, since it breaks the gap down into straight-forward contributions from employment, average hours and labour productivity deviations from trend. The chart shows that the estimate of the output gap is currently made up of a negative contribution from employment and a smaller contribution from labour productivity lying below their potential levels, partly offset by a positive contribution from average hours exceeding its long-run downward trend. More generally, this approach is consistent with the employment gap driving much of the cyclical variation of output with smaller contributions from other sources.

Chart 5.3: Gap contributions (I)

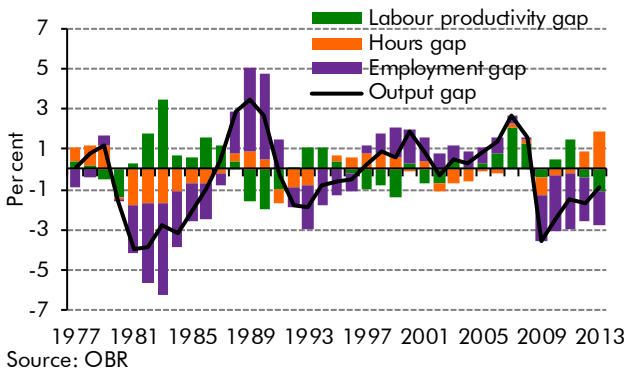
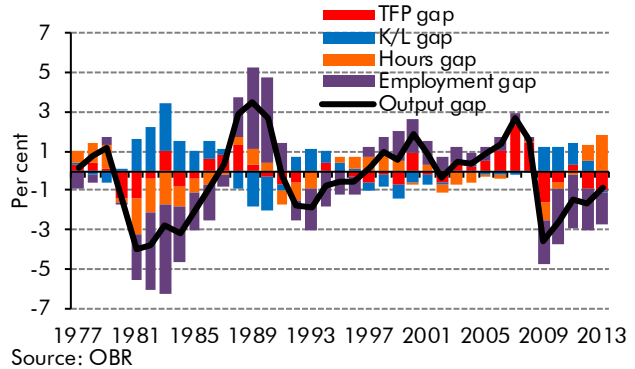


Chart 5.4: Gap contributions (II)



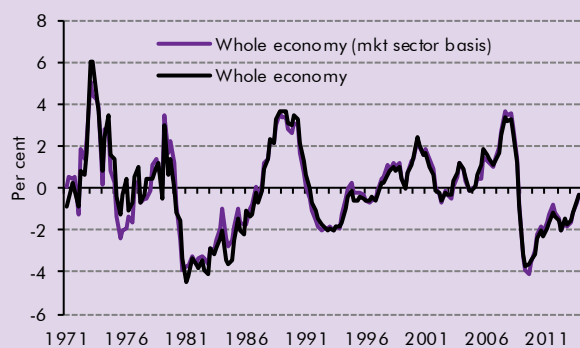
5.12 Another, more detailed decomposition gives us an indication of the TFP and capital intensity contributions to the output gap. This method shows that much of the current labour productivity gap is accounted for by TFP falling short of its potential level. Capital per worker (K/L) appears now to be making a neutral contribution to the output gap, having contributed positively since the recession started.

### Box 5.1: Taking account of market sector information

Many of the traditional methods used to estimate the output gap are based on inferring supply from the balance of demand. But demand is really a market sector concept, so it would make sense to estimate the output gap on this basis. In what follows, it is shown that variations in the output of the non-marketed sector of the economy contribute to output gap volatility when measured on a whole-economy basis. This is best investigated using the production function approach.

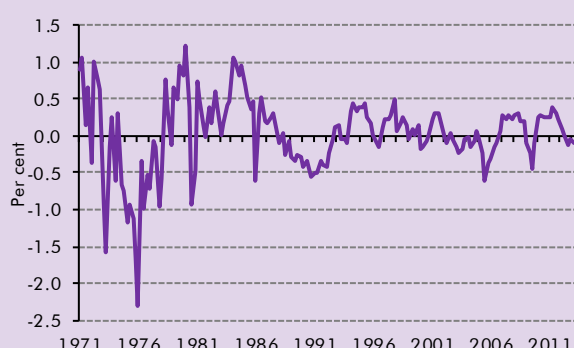
To arrive at a whole-economy equivalent measure for the output gap, non-marketed TFP, labour input and capital are assumed to be at their trend values at all times. Actual values for market sector unemployment, hours and activity (labour input) are combined using the production function methodology with estimates of the capital stock and market-sector, non-oil GVA to arrive at series for market sector TFP. These series are then de-trended to arrive at an estimate of the market sector output gap.

Chart D: Output gaps



Source: OBR

Chart E: Differences



Source: OBR

Charts D and E show that using market sector data does affect estimates of the whole-economy output gap. This is largely because it excludes any information from the government sector, which, theoretically, should not be cyclical, and avoids the issues associated with the measurement of its output.<sup>9</sup> The differences are, though, typically small, particularly over the past 25 years.

Chart F shows that the level of actual TFP in the market sector fell by substantially more in the most recent recession than in the economy as a whole, which is to be expected because of the way in which public sector productivity is measured – market sector TFP fell nearly 8½ per cent from peak to trough, compared with around 6 per cent in the economy as a whole. It also fell more sharply in 2012 than did whole economy productivity. For this reason, trend (filtered) estimates of market sector TFP appear to continue falling, long after trend whole economy TFP stabilises.

Chart F: Actual TFP

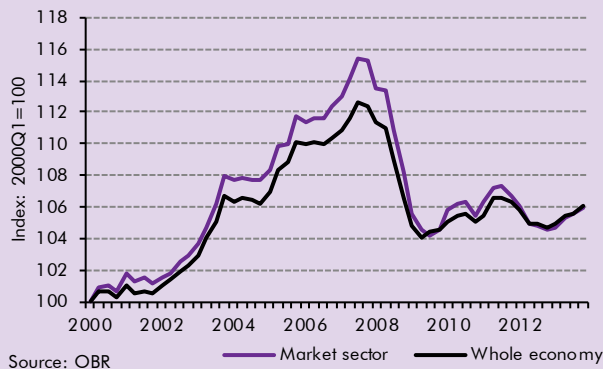
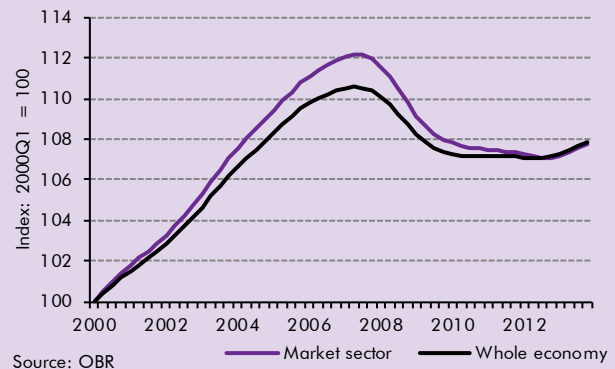


Chart G: Potential TFP



The conclusion to draw from this is that the way in which public sector productivity is measured could muddy the picture of the process we really want to examine – it is the growth of market sector TFP which matters most for projections of receipts, for example. But the price paid for this more disaggregated information is greater complexity and dependency on the assumptions required to construct a market sector data set.

<sup>a</sup> See Atkinson (2005) for a full discussion of the challenges associated with measuring government output in the UK.

5.13 The production function can also be used to infer what other methods of output gap estimation say about the level of potential TFP. In what follows, I begin by taking the level of potential output associated with each method. I then subtract the contribution of potential labour and capital input used to construct the production function to arrive at an estimate of potential TFP.<sup>17</sup>

Chart 5.5: Potential TFP level

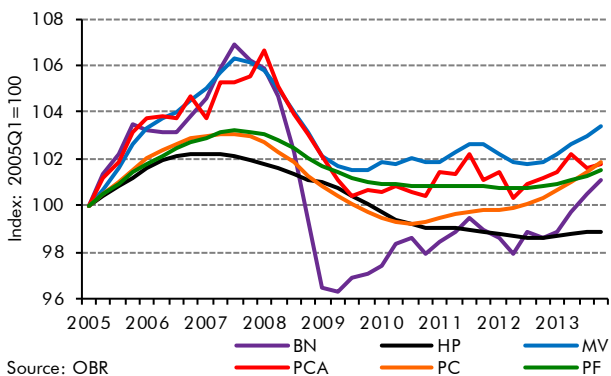
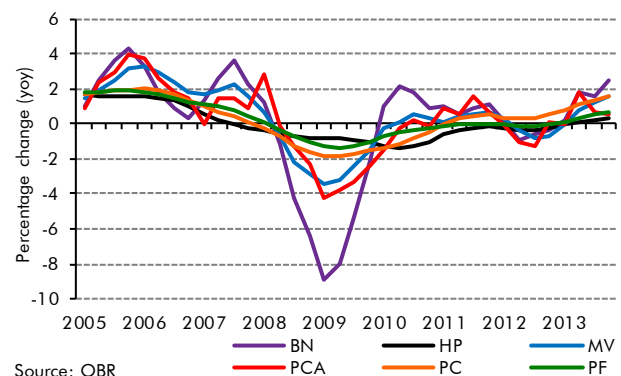


Chart 5.6: Potential TFP growth



5.14 Chart 5.5 shows that each of the HP, MV, PCA, PC and production function (PF) methods is consistent with a significant and permanent fall in the level of potential TFP over the crisis period (although the HP filter dates the start of the slowdown as pre-crisis). Interesting too, is that most of the methods are unable to explain the weakness of productivity over 2012,

<sup>17</sup> To facilitate this comparison, the production function and labour-market augmented filters use the same estimate of the NAWRU.



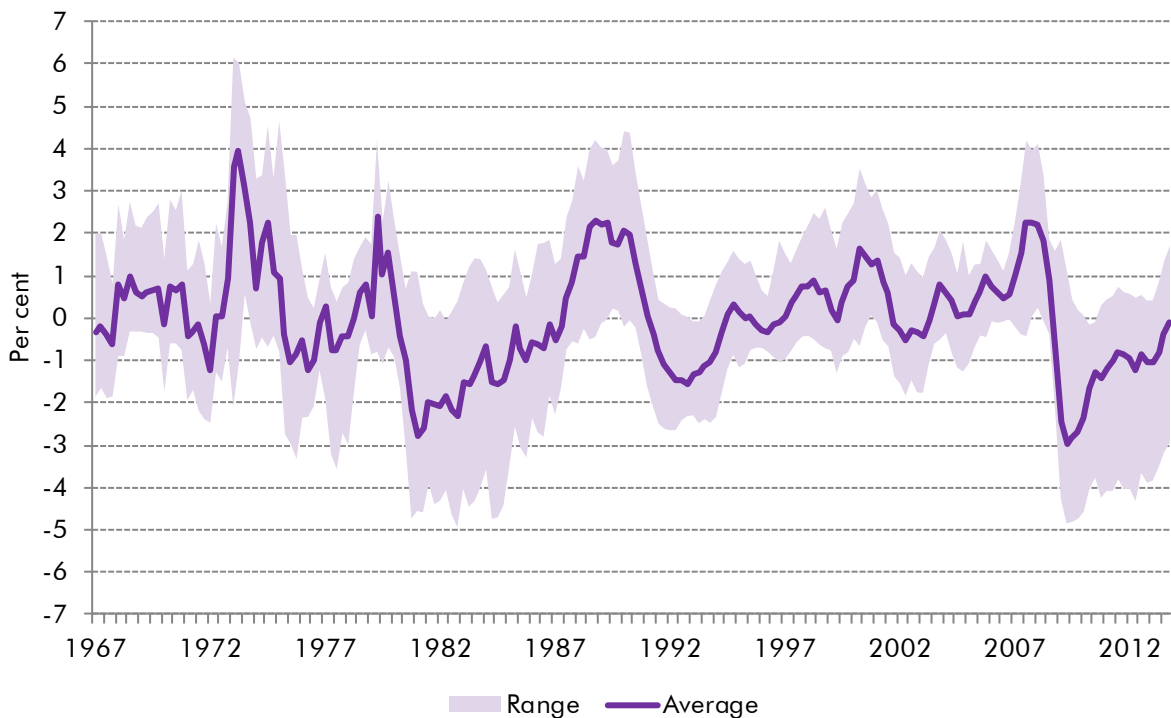
## Estimating the output gap: production function approach

instead, attributing it to weaker potential TFP growth – Chart 5.6. While breaking production function estimates of the output gap into factor contributions, and considering the TFP implications of other estimation methods, may give some insight into how the output gap might have evolved, the results should be considered in the context of the wider uncertainties surrounding output gap estimation.

## 6 Summary of estimates

- 6.1 This chapter summarises the estimates presented in the preceding chapters. The many model assumptions and possible judgements required to estimate the output gap are borne out in the wide range of estimates produced – illustrated in Chart 6.1.<sup>18</sup> And, of course, one might expect this range to expand with the inclusion of other methods.
- 6.2 The range of output gap estimates is around 4.7 percentage points over 2013 as a whole, which compares with a range of estimates made by external analysts at the end of 2013 of 5.7 percentage points.<sup>19</sup> There must, therefore, be a more diverse population of definitions, methods and judgements applied by other analysts than is presented here, indicating that it is worth considering a broader range of evidence when reaching a judgement on spare capacity.

Chart 6.1: Swathe of output gap estimates



- 6.3 It has been suggested that aggregating estimates from a range of models may reduce sensitivity of the measures to model-specific bias – Armstrong (2001). The mean of model estimates is presented in Chart 6.1, but it is worth considering that this, in some sense, is arbitrary because it places an equal weight on each measure and there is no reason to suppose each is equally plausible. Nonetheless, the chart shows that the range of

<sup>18</sup> For the purposes of constructing the chart, estimates from the linear trend methodology are excluded.

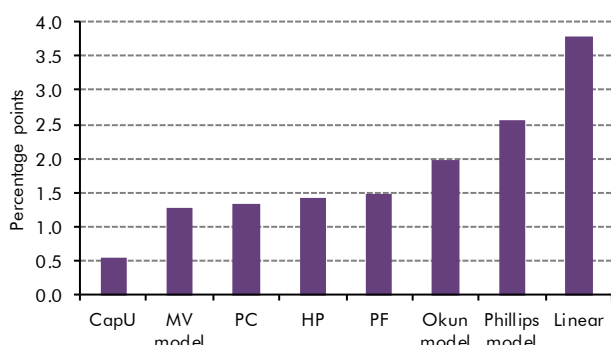
<sup>19</sup> See HM Treasury (2013).

## Summary of estimates

uncertainty varies substantially over the sample period and that the average of estimates rarely lies in the centre of that distribution. Alternative methods of aggregation, such as Bayesian model averaging, may provide more insight into model uncertainty, but I have not explored that in this paper.

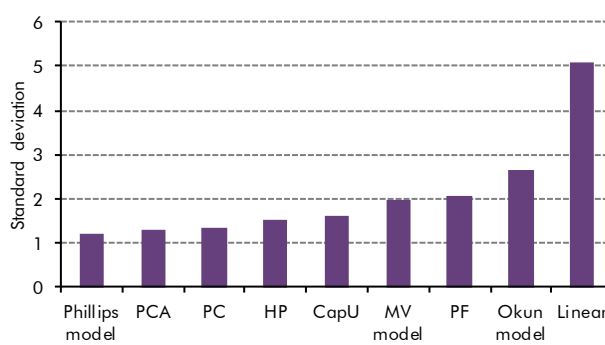
- 6.4 Chart 6.2 illustrates the tendency of each measure to be revised – as stated in Box 6.1, this should not be the only criteria one considers when assessing the performance of output gaps. The chart shows that there is substantial variability in the revisions properties of the models presented, with estimates generated by linear de-trending much more likely to be revised than the multivariate filter, for example. Likewise, the volatility of the output gap and potential growth vary significantly between methods – Charts 6.3 and 6.4.

Chart 6.2: Revisions



Source: OBR

Chart 6.3: Volatility of output gap



Source: OBR

- 6.5 Chart 6.5 shows that, generally speaking, the lower the standard deviation of potential output growth, the more prone is an output gap measure to revision. Linear de-trending, for example, lies in the south-east corner – it assumes a constant potential output growth rate, but its output gap estimates are revised the most. Another way of thinking about it is that a linear growth model places the most weight on information from the future (the whole sample determines the slope of the line) in determining its estimates of the output gap. So, naturally, as more data become available it makes big revisions to its earlier estimates. Other methods place less weight on future information and, therefore, they tend to be revised less.<sup>20</sup>

<sup>20</sup> The statistics for the Phillips Curve method are over a sample from 1995-2013, as uncertainty over the steady state of inflation in the earlier years of the sample causes significant revisions that are not considered relevant to this comparison.

Chart 6.4: Potential output growth volatility

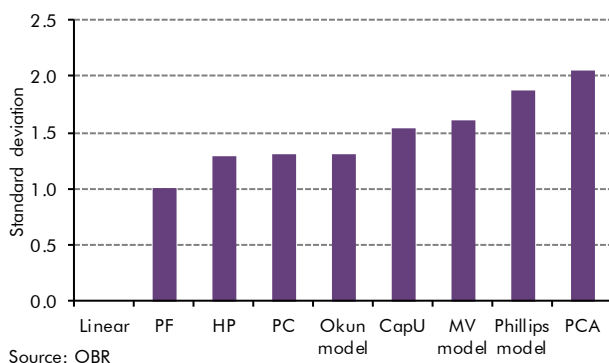
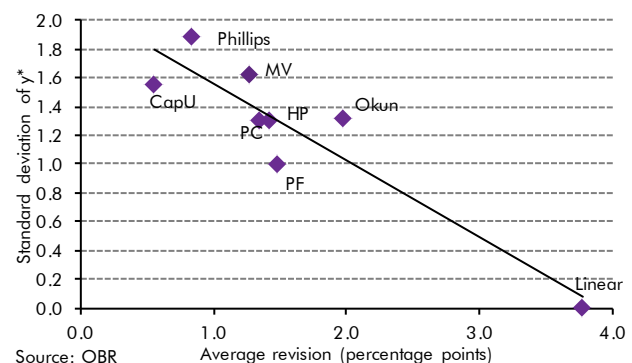


Chart 6.5: Revisions vs potential output volatility



### Box 6.1: How smooth is potential output?

A common feature of all the output gap estimation methods presented here is that they are underpinned by assumptions, explicitly or implicitly, about the smoothness of potential output. The Hodrick-Prescott (HP) and prior-constrained (PC) filters, for example, require explicit judgements over the choice of smoothing parameter.

The amplitude of the cycle obtained using principal components depends on the choice of scaling parameter, which determines the medium-term variation in potential output. Directly estimating the output gap with principal components also allows very high frequency movements in the survey data and measurement error to feed through to the level of potential.

The production function approach allows the user to apply more disaggregated judgements to arrive at an overall smoothness. For example, the structural rate of unemployment, which depends on things like the degree of wage indexation to inflation and union intensity, might be less variable than potential total factor productivity (TFP) over time.

Yet, once decisions have been made about the smoothness of the other components of potential output, one is left with the crucial judgement over the smoothness of potential TFP. Unfortunately, the economics profession has little to say about the processes that determine it, which makes judgements about its potential level, volatility and likely growth rate dependent on historical averages – the key judgement being how much weight to place on the recent past relative to longer time periods.

For example, most economic commentators viewed the sustained period of strong productivity growth in the UK over the 2000s as being structural. And, with no theory to suggest otherwise, it was hard to make the opposite argument. During the 2008 financial crisis, TFP fell sharply, and has barely grown in the subsequent six years. This performance has prompted some to revisit their assumptions about the sustainability of growth in the preceding years. This is illustrated in Chart H, which shows the output gap path the OECD published before the crisis, in 2007, and the one it published in 2014. And to a lesser extent in Chart I, which shows the Treasury and, latterly, OBR estimates.

Chart H: OECD output gaps

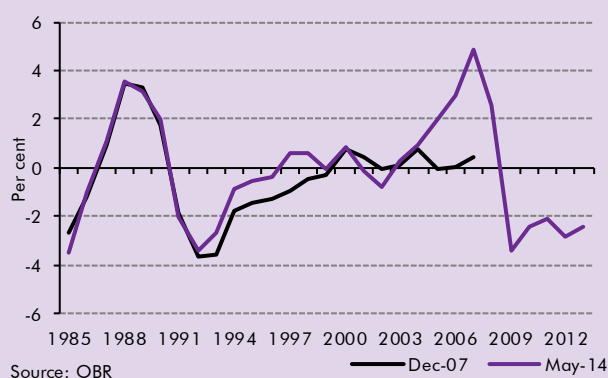
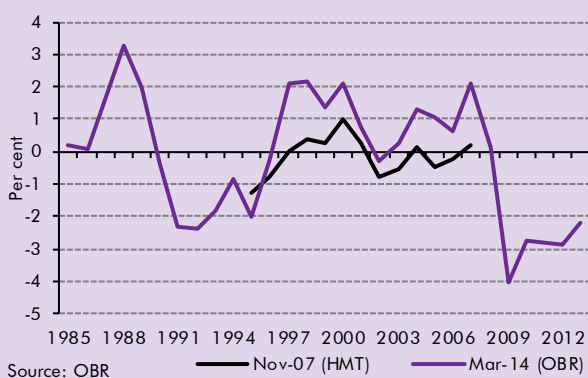


Chart I: Treasury, OBR output gaps



The OECD, which uses a production function method, estimated with a two-sided filter, now judges that there was a significant positive output gap in 2007 of around 5 per cent. This interpretation is consistent with potential TFP growth having slowed substantially in the years before the crisis started. Partly, this reflects the properties of the HP filter, which penalises sharp movements in potential output. An alternative interpretation, from the multivariate (MV) filter or principal components analysis (PCA), for example, is that potential TFP continued to grow at roughly its usual rate up until the crisis began, but fell sharply during it.

So which interpretation seems more plausible? Potential TFP growth is considered to reflect advances in technology and improvements in efficiency (such as better process management, for example). It is difficult to understand how a substantial TFP gap could open up, since it implies excessive utilisation of technology available at the time – working the capital stock at unsustainable rates, for example. Yet there was no evidence of this in the period before the crisis.

But it is also hard to see how the state of technology and efficiency can suddenly regress so far – have we forgotten how to do things we could do previously? Or is it that some of the things we were doing, which seemed to make things more efficient at the time, turned out to be less useful than we thought (some activities of the shadow banking sector, for example)? All we really have to go in is that actual TFP fell and has barely grown for six years and either the gap or level hypothesis could be correct.

So, it is important to recognise that different methods for estimating the output gap are consistent with different interpretations of the process driving TFP. The implication is that output gaps should not just be ranked on their tendency for revision: a method that assumes potential TFP can fall sharply will tend to be revised by less than a method that assumes it cannot. Given the scant theory on the subject, there is no strong case for favouring one view of the world over the other, so both should be considered when making a judgement and neither discarded on the basis of tendency for revision.

- 6.6 The methods presented above are by no means intended to capture all the methods of output gap estimation available to forecasters, but it does capture a wide range, which is reflected in the variability of estimates over time. There are other methods not considered here, such as the multivariate Kalman filter, as applied by Konuki (2008). In practice, this is very similar to the MV (PC) filter, presented above. The parameters of a multivariate Kalman model must be calibrated, often under the assumption that they are deep parameters – i.e. stable and based on microeconomic evidence. The PC filter also depends upon calibration and some of the parameters could be considered reduced-form representations of structural relationships.
- 6.7 The MV (PC) filter requires explicit judgements over smoothing parameters and the weights placed on explaining the path of potential output, while a Kalman model requires explicit priors over the size of shocks to its measurement equations. Likewise, estimates produced using models that utilise Bayesian methods – such as Benes *et al* (2010) – require priors over the size of model parameters and shocks as well as their underlying distributions. Fundamentally, since potential output is a notion constructed by economists, all methods used to estimate it need to be told how volatile it is – there is no escaping this key judgement, which is the subject of Box 6.1.

#### Box 6.2: What is more useful, more accurate information or the benefit of hindsight?

It is a well-documented finding that optimal ex-post monetary policy prescriptions appear different from those made in real time and that this mostly reflects the unreliability of output gap estimates in real time – see Orphanides & van Norden (2002). In these papers, the authors assess the revisions properties of a range of output gap estimates for the United States, including linear de-trending, the HP filter, an Okun relation and an unobserved components model augmented to include the Phillips curve. Across all measures, the role of data revisions is found to be relatively small in explaining policy errors, when compared with those arising from the end-point problem.

Massimiliano & Musso (2011) find that output gap revisions for euro area economies are substantial and of equal magnitude to the output gap estimates themselves – they find that data revisions account only for a small part of this. While Cayen and van Norden (2005) find that revisions to output gap estimates for Canada are significant but can mainly be attributed to data revisions.

To see which of these findings hold for the UK, I assess the revisions to HP filter estimates of the output gap from two sources – new data and revised data. I start by assessing the sensitivity of output gap estimates to data revisions. To this end, I apply an HP filter to both the real-time GDP data and the current vintage and compare the results. The average absolute output gap revision arising from GDP data revisions from the first quarter of 1978 to the final quarter of 2011 is around 0.7 percentage points.

I then compare the one- and two-sided HP filter estimates applied to the current vintage of GDP data – the average revision owing to the arrival of new data (which I call the benefit of hindsight

here, although that would also include updates to model parameters, for example) is found to be around 1.4 percentage points. The influence of data revisions on output gap estimates is presented in Chart J, while the sources of output gap revision are illustrated in Chart K.

Chart J: HP output gap estimates

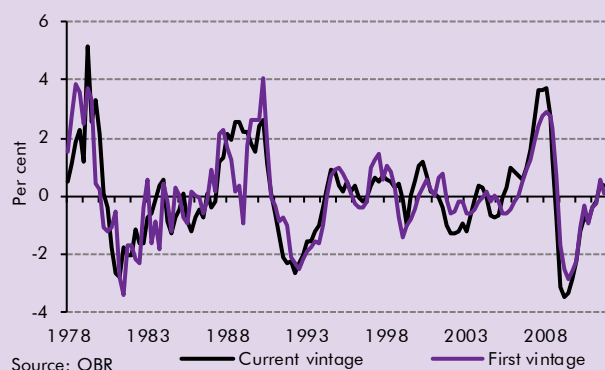
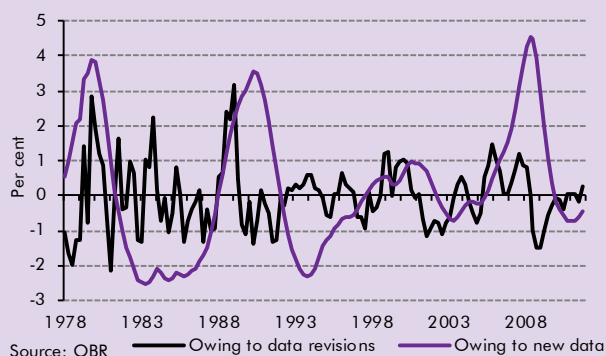


Chart K: Output gap revisions



The results suggest that, on average, the benefit of hindsight is about twice as useful as better data when it comes to improving HP-filtered estimates of the output gap – although this conclusion would probably be affected by the choice of estimation method. The results obtained here lie between those for Canada, where output gap revisions are mostly attributable to data revisions, and the US and euro area, where data revisions play a much smaller role. It is unclear why data revisions should play a different role in UK output gap revisions than in other countries. UK GDP prior to the recent financial crisis now look a bit stronger than early vintages implied, and the recession a bit deeper. To the extent that revisions like this are not features of the data in other countries, the proportion of output gap revisions owing to data revisions will differ from the UK.

Of course, history will continue to be rewritten as methodological improvements come to change our understanding of the past. A major set of National Accounts revisions is due later this summer. And, given that the output gap estimates presented in this paper are based on the current vintage of data only, it is worth keeping in mind that revisions to ONS output data are an additional source of output gap uncertainty.

# 7 The cyclically adjusted fiscal position

7.1 The UK Government sets fiscal policy such that a measure of balance is restored to the public finances over a rolling five-year horizon. In doing so it takes into account the effect of the cycle on revenues and spending, which requires an estimate of the output gap. In practice, every forecast of GDP is underpinned by an assessment (implicit or explicit) of spare capacity and, therefore, how much scope there is for above-trend growth as it is taken up. This section considers the sensitivity of cyclically-adjusted measures of the fiscal aggregates to output gap mismeasurement.

7.2 Recent years have seen the emergence of new literature concerned with the reliability of cyclically-adjusted measures of fiscal aggregates. As reported in Tereanu *et al* (2014) a number of papers have assessed output gap uncertainty and some explore the implications for measures of the public finances:

- Koske and Pain (2008) find that revised output gap estimates account for cyclically-adjusted public borrowing revisions of around 0.4 percentage points, on average, across a range of OECD countries;
- Bouis *et al* (2012) find that output gap revisions average 1 to 1.5 percentage points in OECD countries, but that underlying fiscal balances are not very sensitive to this;
- Hallett *et al* (2009) find that revisions to cyclically adjusted public borrowing estimates owing to output gap revisions average around 1 percentage point in most euro area economies; and
- Ley and Misch (2013) find that revisions have substantial effects on measures of the structural balance.

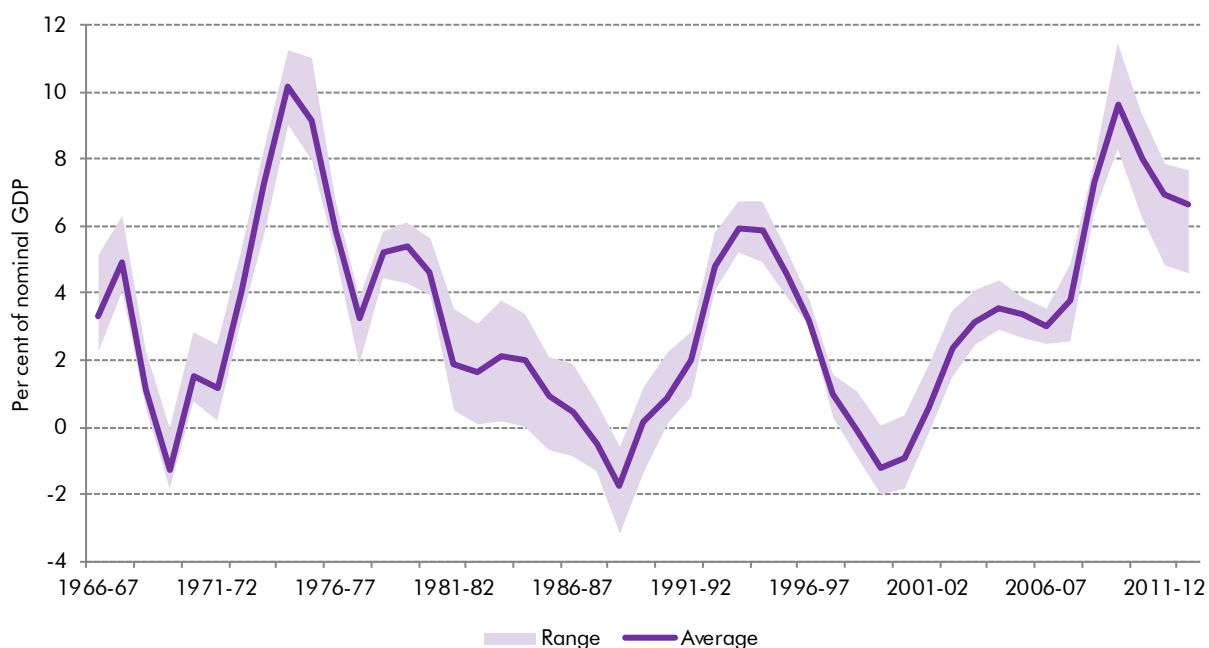
7.3 In what follows, I consider the measurement of cyclically adjusted public sector net borrowing (CAPSNB). It is first perhaps useful to set out the wide band of uncertainty surrounding CAPSNB – this is illustrated in Chart 7.1, which presents estimates based on the suite of output gap measures presented in this paper and ex-post data. Uncertainty over point estimates of the output gap carries over to CAPSNB via the cyclical-adjustment coefficients, roughly 0.7 for 1.<sup>21</sup>

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<sup>21</sup> See Helgadottir *et al* (2012)



Chart 7.1: Swathe of CAPSNB measures<sup>22</sup>



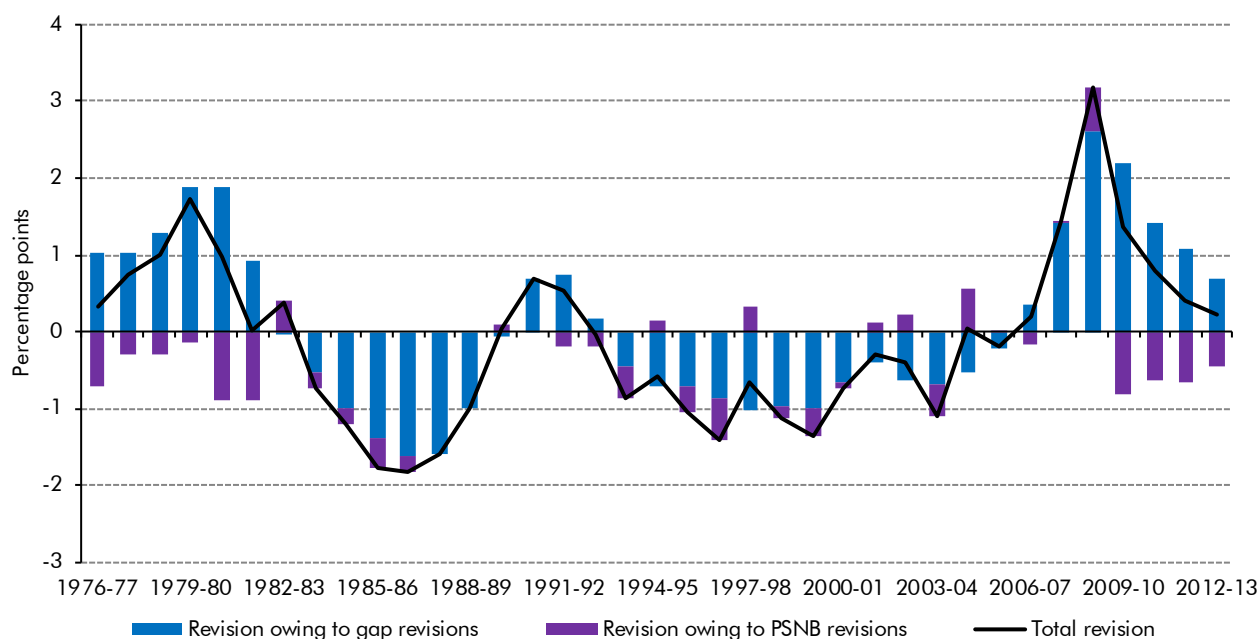
7.4 Uncertainty over point estimates of the CAPSNB is compounded by uncertainty over how estimates might change in future. CAPSNB can be revised for three reasons, of which this section considers the first two:

- the public sector finances and output data can be revised;
- estimates of the output gap can change; and
- estimates of the sensitivity of the public finances to the cycle can be updated.

7.5 So what might be the greater source of uncertainty? To assess this, I first estimate the CAPSNB using initial estimates of PSNB as a share of GDP and a real-time measure of the output gap – in this case given by the one-sided PC filter. This serves as a baseline case. To assess sensitivity to output gap revisions, I see how the estimates change when I calculate the CAPSNB using an ex-post output gap (two-sided PC filter). Finally, I calculate the CAPSNB using the latest fiscal data and an ex-post output gap, to capture the effect of data revisions. The results are presented in Chart 7.2.

<sup>22</sup> Data are cyclically-adjusted PSNB from 1994-95, and the cyclically adjusted public sector borrowing requirement before that.

Chart 7.2: CAPSNB revisions



Source: OBR

- 7.6 The purple bars in Chart 7.2 can be interpreted as total revision owing to revised fiscal data (and in-year forecast errors), while the blue bars are revision because the output gap path is now thought to be different – the purple and blue bars sum to the total CAPSNB revision. The chart shows that the estimates of structural borrowing through the financial crisis presented here are revised much higher once data revisions and revised output gap paths are taken into account – with the latter playing the dominant role. This is consistent with the findings of Tereanu *et al*, who state that “during the crisis years, the estimates of CAPB were considerably worse for most countries after they were re-estimated following budget execution.”
- 7.7 On average, the magnitude of revisions to CAPSNB owing to output gap revisions is around 1 percentage point, far larger than the average revision arising from revised data of 0.3 percentage points.<sup>23</sup> So output gap uncertainty is likely to be the larger source of uncertainty over the structural fiscal position in real time, although selecting other measures of the output gap with different revisions properties could affect this conclusion. This result is consistent with the findings of Hallett *et al* (2009), who find that CAPB revisions owing to output gap revisions are around 1 percentage point on average.
- 7.8 The average output gap revision across the range of methods applied to UK data earlier is around 1.3 percentage points. This is roughly consistent with the findings of Tereanu *et al*, who find that the average revision for EU countries is around 1.5 percentage points. Using cyclical adjustment coefficients summing to 0.7, it would be reasonable to assume that revisions to CAPSNB would be close to 1 percentage point using that range of methods.

<sup>23</sup> For simplicity, the revision to the output gap does not take into account revisions to the path of GDP so, as is shown in Box 6.2, output gap uncertainty may be an even larger source of uncertainty.

7.9 It is worth noting that the analysis above suggests that revisions to the CACB tend to be largest around turning points, also consistent with the conclusion reached by Tereanu *et al.* This is problematic, since these are precisely the moments when policymakers most need them to be reliable.

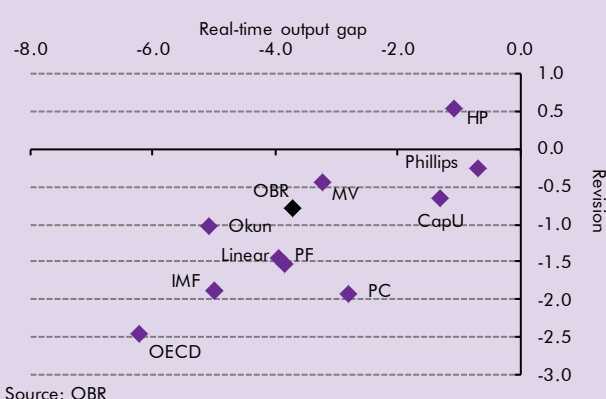
### Box 7.1: The structural fiscal position in 2010 and the benefit of hindsight

In assessing measures of the output gap, it is interesting to look at particular points in economic history to see how the estimates associated with them have changed. Here, I consider the second quarter of 2010, since it was a time when a new Government took office and made significant changes to the planned path of fiscal policy that were informed by the advice it received. Table A sets out the revisions to output gap estimates from a range of output gap estimation methods and includes estimates from forecasters of the UK economy made during 2010 and early 2014 - including the official forecasts of the OBR and those of the OECD and IMF (for the 2010 calendar year).

Table A: Output gap revisions

	Real-time	Ex-post	Revision
Linear	-3.8	-2.3	-1.5
HP	-1.1	-1.6	0.5
PC	-2.8	-0.9	-1.9
Phillips	-0.7	-0.4	-0.3
Okun	-5.1	-4.0	-1.0
CapU	-1.3	-0.6	-0.7
MV	-3.2	-2.8	-0.5
PF	-3.9	-2.5	-1.5
OBR	-3.7	-2.9	-0.8
OECD	-6.2	-2.5	-3.7
IMF	-5.0	-1.9	-3.1

Chart L: Revisions and output gap size



Source: OBR

The results show that all measures were revised, with an average revision of around -1.3 percentage points. It is also the case that methods consistent with larger output gaps to begin with were revised by more – this is likely to reflect the fact that methods that assume a less volatile path for potential output (and therefore larger amplitude output gaps) are more likely to place weight on future data to inform that trend and be revised accordingly.

The results reported in Table A for the international organisations probably illustrate the uncertainty associated with hindsight about the data and about the model. For example, the IMF’s estimate of the negative output gap in 2010 fell from 5.0 per cent in its April 2010 WEO to 2.7 per cent in its October 2010 WEO. Its estimate of the positive pre-crisis output gap in 2007 was 0.4 per cent in the April 2010 WEO and 1.0 per cent in the April 2011 WEO. It was not until the October 2012 WEO that the IMF started to estimate a very large positive output gap in that year, of 3.7 per cent.

The benefit of hindsight generally tells us that the output gap was narrower in 2010 than was thought at the time and so the structural fiscal deficit was larger.

7.10 This section has shown that output gap mismeasurement is a significant source of uncertainty over the real-time fiscal position and probably a larger one than revisions to the public finances data. But it is worth remembering that, when fiscal policy objectives are set in the future, the output gap is only one part of what is needed to forecast the cyclically-adjusted fiscal position. One also needs to make a judgement about how fast the economy might be able to grow on a sustainable basis. This paper is not concerned with that issue, although it is likely to be equally, if not more, important.

## 8 Conclusion

- 8.1 This paper began by emphasising that the policy horizon should be taken into account when formulating the definition of the output gap. This reflects the possibility that fiscal authorities and central banks may have different perspectives, particularly when output gaps are large. Output gap estimates are prone to revision, not least because the output gap is a concept invented by economists that cannot be observed, only estimated. Three sources of output gap revision were then identified – those arising from the arrival of new data, revisions to past data and changes to model specification.
- 8.2 It has been shown that revisions owing to the arrival of new data are large and, on average, tend to be of the same magnitude as the output gap estimates themselves. Output gap revisions owing to revised data are also likely to be significant, while the range of output gap estimates produced by the handful of methods presented here is shown to be substantial. Overall, the level of uncertainty about the size of the output gap is high and it is shown that this carries over to estimates of the cyclically adjusted fiscal position, while revisions to public sector finances data also contribute to a smaller degree.
- 8.3 Along the way, it has been illustrated that the assumptions underpinning various methods for estimating the output gap are consistent with different views of how the economy functions – in particular, about the time series properties of potential productivity, about which little is known. Whether the views underpinning a methodology are explicit or implicit, they represent the application of judgement to the estimation of spare capacity. No methodology can be made totally free from judgement.
- 8.4 It has also been shown that the assumptions underpinning some methods mean they are more likely to be revised and, while other methods are less prone to changes, they may not be any more useful in forecasting the path of actual GDP – so no method should be discarded based only on its tendency to be revised.
- 8.5 The implication of the analysis presented here is that no single method for estimating the output gap is likely to be reliable at all times and a wide range of evidence should be considered when reaching a judgement about spare capacity.

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## B Pre-filtered variables

### Estimating the structural rate of unemployment

8.8 Some of the methods presented in this paper make use of the structural rate of unemployment to estimate the output gap. To ensure consistency between those methods, I estimate the structural rate of unemployment in advance of output gap estimation. To do this, I assume that the structural rate is as a random walk without drift and apply the PC filter to it. I augment that filter to include information from real wages, adjusted for underlying movements in productivity growth – the estimated structural rate can, therefore, be thought of as a long-term non-accelerating wages rate of unemployment (NAWRU).

The long-run NAWRU is estimated in state space using the Kalman filter and is given by Equations 44 to 50:

$$\text{Signal: } u_t = u_t^* + uc_t \quad (44)$$

$$\text{State: } u_t^* = u_{t-1}^* + \left( \frac{\varepsilon_{1,t}}{k^{1/2}} \right) \quad (45)$$

$$\text{State: } uc_t = \varepsilon_{1,t} \quad (46)$$

$$\text{Signal: } w_t = prod_t^* + \beta_5 uc_t + \beta_6 uc_{t-1} + \varepsilon_{5,t} \quad (47)$$

$$\text{Signal: } prod_t = prod_t^* + prodc_t \quad (48)$$

$$\text{State: } prod_t^* = prod_{t-1}^* + \left( \frac{\varepsilon_{1,t}}{\varphi_{prod}^{1/2}} \right) \quad (49)$$

$$\text{State: } prodc_t = \varepsilon_{1,t} \quad (50)$$

8.9 Equation 47 posits a simple relationship between the growth of real product wages,  $w_t$ , as measured by average earnings deflated by the gross value added deflator at factor cost, the unemployment rate gap and a measure of trend productivity growth – to capture underlying movements in labour productivity (unrelated to the cycle) that could influence wages. The labour productivity trend is jointly estimated with the NAWRU. The coefficients are calibrated drawing from evidence, including Greenslade *et al* (2003).

Chart 8.1: Real-time NAWRUs

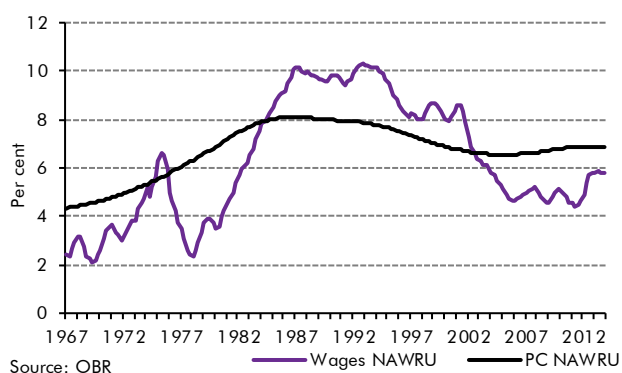
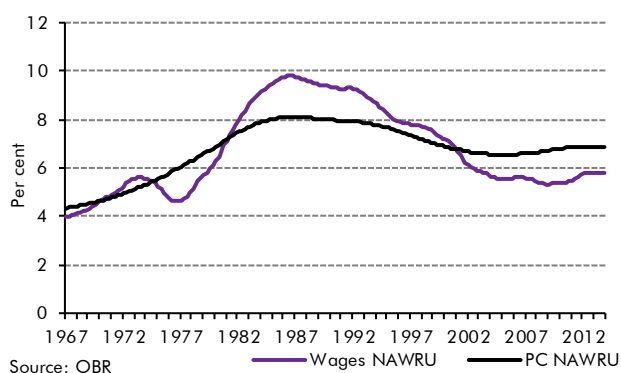


Chart 8.2: Ex-post NAWRUs



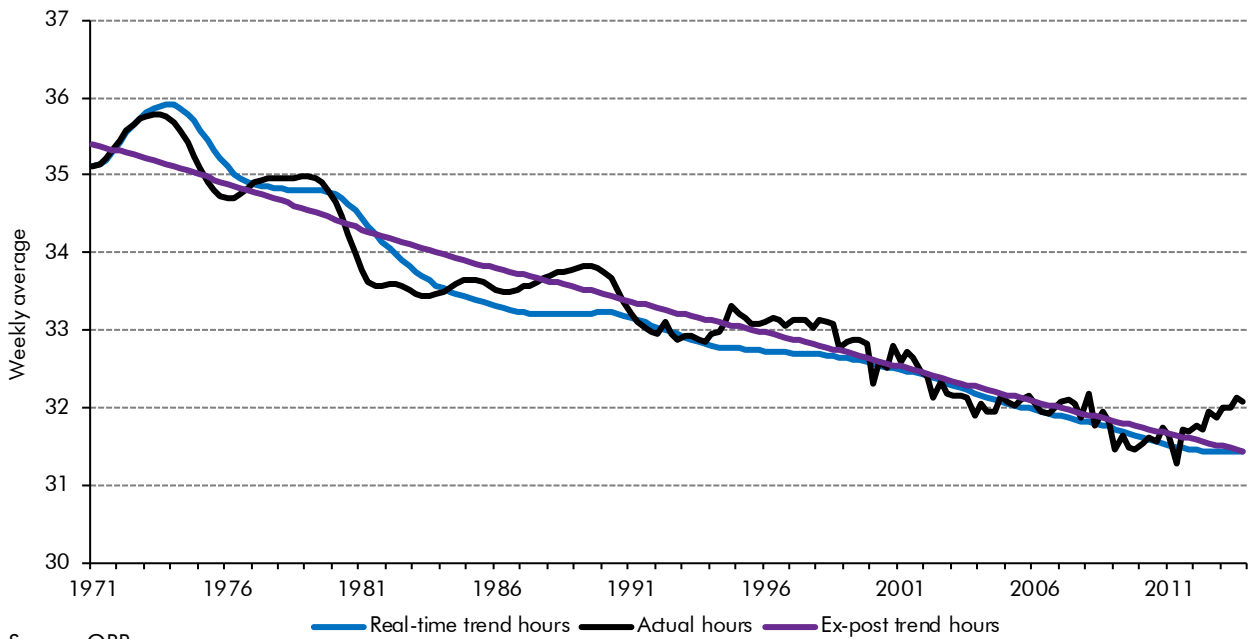
8.10 Charts 8.1 and 8.2 show the effect of including wages on the NAWRU estimates, relative to using the naïve univariate PC filter. Because real wages growth was low and stable in the 2000s, the PC filter is able to identify that the NAWRU must have fallen in the preceding years. Likewise, the weakness of real product wages growth during and following the recession helps the filter to identify more cyclical weakness in unemployment, preventing a significant increase in the long-run NAWRU estimate.

8.11 This, combined with the stiffness of the filter, is consistent with the view that little has changed in the structure of the labour market over that period, and that the long-term unemployed will eventually find their way back into employment, for example.

## Estimating potential average hours

8.12 Average hours have drifted downwards for centuries, as productivity has risen and additional income has been substituted for more leisure time. To model trend average hours, I apply the PC filter with drift to the average hours series using a stiff smoothing parameter. The real-time and ex-post results are illustrated in Chart 8.3. A key uncertainty is the extent to which the latest increase in average hours reflects a permanent response to a loss of permanent income relative to prior expectations. The estimate presented here is consistent with average hours eventually falling back to the long-run trend.

Chart 8.3: Estimates of potential average hours

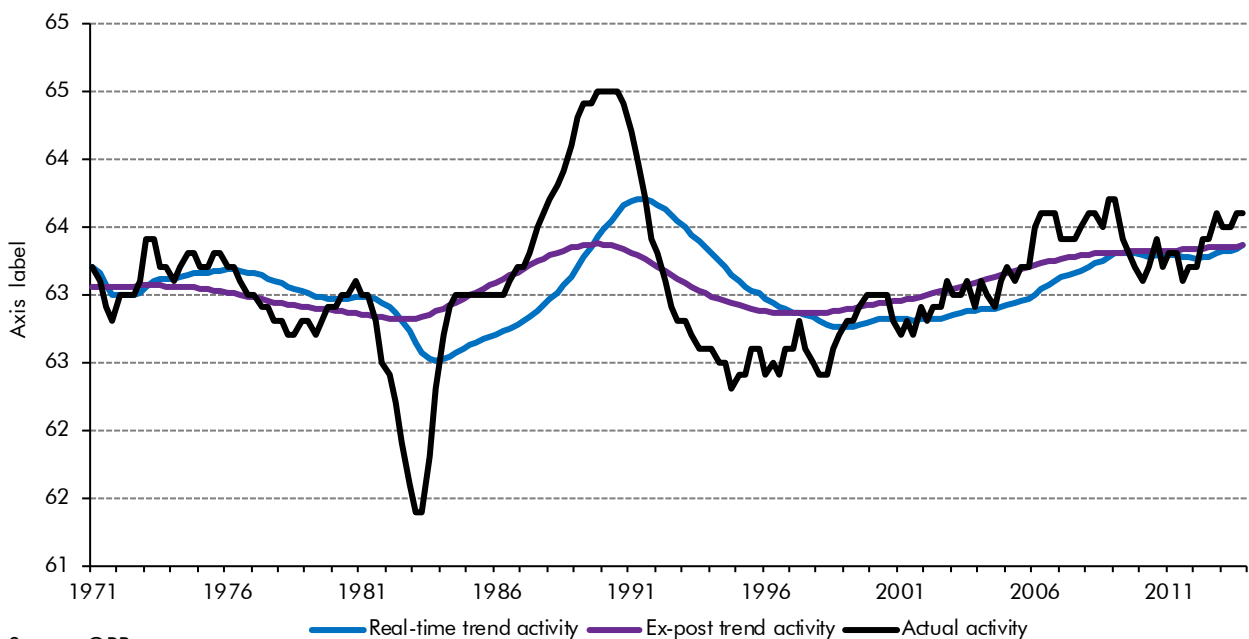


Source: OBR

## Estimating potential activity

8.13 Potential activity has also been estimated using the PC filter but, unlike average hours, without a drift term – the estimates are presented in Chart 8.4 Further information relating to demographic influences on participation rates could be introduced via a cohort model, for example, but that is not explored here.

Chart 8.4: Estimates of the potential activity rate



Source: OBR

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