The impact of public investment on private investment: comment on 'Public investment and potential output', OBR Discussion paper No.5.

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Introduction

OBR5[2] makes an important contribution to understanding the long run effects of public investment. This comment argues that it is important to include the effect of public capital on private investment, and to model the complementarity/ substitutability between the two. Using the framework and parameters of OBR5 the comment shows that this can have large effects on the impact of public investment.

The central arguments in OBR5 are: (a) A 1 percentage point increase in the share of GDP going to public investment will raise the long run public capital stock by 27%. (b) A 27% increase in the public capital stock will raise GDP by 2.4 % (holding constant other inputs). (c) This yields a rate of return on public investment of 8.7%.

The focus of this comment is on point (b). Using the framework and parameters of OBR5 the comment establishes possible responses of private investment to a higher public capital stock, and the further changes in GDP that this implies. For example, if there is quite strong complementarity between the two types of capital the impact on GDP is nearly twice that suggested in OBR5. The comment also reports impacts on real consumption, undertaking a welfare analysis in which costs of investment are netted out from additional income.

On point (a) we merely note that the 27% increase in public capital stock seems small. A naïve calculation suggests that raising public investment from its historical range of 2 - 2.5% of GDP to 3 - 3.5% might be expected to raise the steady state capital stock by 40 or 50% (3.5/2.5 = 1.4).

The private investment response to public capital:

The following remarks are based on steady state relationships, entirely consistent with the OBR5 long-run model. Exactly as in OBR5 a Cobb-Douglas (CD) production function with elasticity of output with respect to public capital (K_G) equal to 0.1 gives output response $1.27^{0.1} = 1.024$. As it does so it raises the return on private capital, K_P, (marginal product MPK_P) by the same proportion.

This may trigger a private response. To quantify this, suppose that private investment increases the private capital stock, K_P, to the point at which its marginal product, MPK_P, returns to the base level. With OBR5 parameters (in particular, elasticity of output *Y* with respect to private capital K_P = 0.23) the steady state private capital stock increases by 3% and, taking the public and private changes together, income rises by 3.2%, nearly half as much again as in OBR5 where K_P is held constant. In OBR5 this mechanism is assumed not to operate. (It is mentioned in paras 5.7-5.10).

The sign and magnitude of this response depends critically on the elasticity of substitution between public and private capital. Suppose that instead of CD the production function is CES between the two types of capital, remaining CD between this capital aggregate and the other input, labour, L, so

$$Y = L^{1-\alpha-\beta}\left\{\frac{\alpha K_G^{1-\frac{1}{\sigma}} + \beta K_P^{1-\frac{1}{\sigma}}}{\alpha+\beta}\right\}^{\frac{\sigma(\alpha+\beta)}{\sigma-1}} \quad with \quad \frac{\partial Y}{\partial K_p} = L^{1-\alpha-\beta}K_P^{-\frac{1}{\sigma}}\left\{\frac{\alpha K_G^{1-\frac{1}{\sigma}} + \beta K_P^{1-\frac{1}{\sigma}}}{\alpha+\beta}\right\}^{\frac{1+\sigma(\alpha+\beta-1)}{\sigma-1}}.$$

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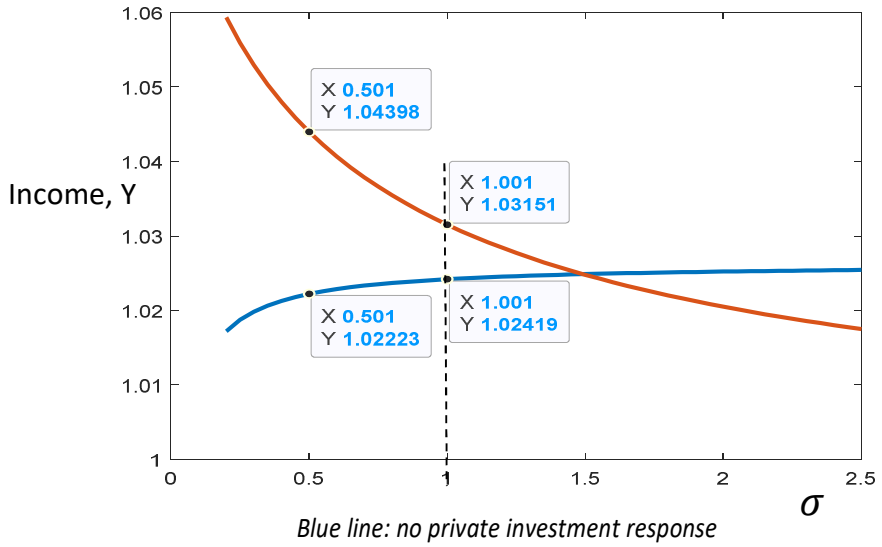
[2] 'Public investment and potential output', August 2023, N. Suresh, R. Ghaw, E. Obeng-Osei, T, Wickstead, Discussion paper no. 5, Office for Budget Responsibility, London, UK

Parameter σ is the elasticity of substitution between K_P and K_G and $\alpha, \beta, 1-\alpha-\beta$ are the elasticities of output with respect to inputs, taking the OBR5 values $\alpha = 0.1,\ \beta = 0.23$. Units of measurement for K_G, K_P, L are set such that in the base they all take value unity. This is without loss of generality and means that, as values of σ are varied, the elasticities of output with respect to each of the inputs remain (in the neighbourhood of the base) equal to OBR5 values, $\alpha = 0.1,\ \beta = 0.23$.[3]

Figure 1 has σ on the horizontal axis, and the change in income associated with a 27% increase in K_G on the vertical. The blue line holds K_P constant, and the red line lets it adjust to restore the private rate of return (MPK_P) to its base value. OBR5 reports the case on the blue line at $\sigma = 1$, giving the output change of 2.419%.

Figure 1: Complementarity vs substitutability: output responses to a 27% increase in public capital



Blue line: no private investment response

Red line: with private investment response

Using the same parameters but letting K_P adjust gives significantly larger output change of 3.151%. The role of complementarity/ substitutability of public and private capital is clear from divergence of the two lines on the figure, with larger effects the stronger the complementarity, i.e. the lower is σ. Thus, at $\sigma = 0.5$ with K_P endogenous output increases by 4.4%, nearly twice the OBR5 number.

Conversely, if σ is sufficiently large then public capital substitutes for private. In that case additional K_G reduces the return to private capital falls and the private response is lower investment, a reduction in K_P. As illustrated on the figure, the dividing line between these cases is at $\sigma \cong 1.5$ (the value depending on values of α, β).[4]

This illustrates the importance of allowing a private investment response, although aggregate analysis of this type can only be illustrative of mechanisms and possibilities. To go to real estimates requires knowing what assets the public sector invests in and the extent to which these assets are likely to trigger private investment (their substitutability/ complementarity to private capital). Some investments are likely to be complementary with private investment (e.g. infrastructure), others may be substitutes (e.g. housing). Some public investments may well trigger processes of cumulative

[3] Without this specification total output and elasticities of output with respect to inputs (the only data used in the model) would vary with σ, even with inputs held constant. This would render comparisons meaningless.
[4] The red and blue lines intersect at the point where MPK_P is independent of K_G, $\partial^2 Y / \partial K_p\, \partial K_G = 0$, i.e. where $\sigma = 1/(1-\alpha-\beta)$, see equations.

causation, agglomeration and cluster formation. This adds a further benefit as increasing returns to scale raises productivity, raising output levels at each level of inputs.

Note that this analysis assumes a perfectly elastic supply of capital, as does OBR5.

Real consumption effects:

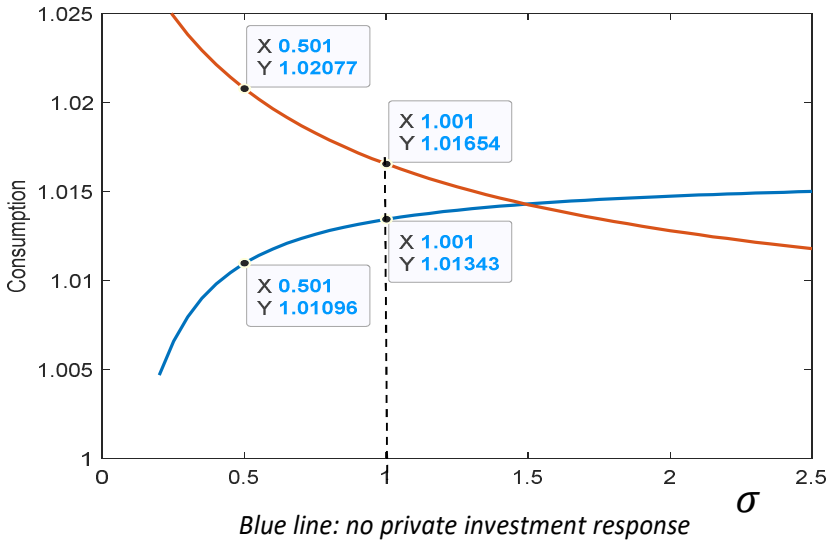
Investment has a real cost – of foregone consumption – which needs to be considered in developing a true picture of effects. Applying principles of basic welfare economics, we know that if public and private investment in the base are at efficient levels (i.e. levels at which the rates of return on each type of capital equal the marginal social cost of capital, MSCC), then a further increase in a capital stock will reduce real consumption and welfare – it is a move away from efficiency. Conversely, if capital stocks are initially sub-optimal (yielding returns above the MSCC), then there are efficiency gains and an increase in real consumption.

This can be captured by placing a 'wedge' between returns on capital and the MSCC. Figure 2 imposes wedges on both private and public capital, with the base level of public capital earning twice the MSCC and private earning 1.5 times.[5] The figure has the same structure as Figure 1, but with consumption (relative to base) on the vertical axis; at $\sigma = 1$ the increase in real consumption with private sector response is 1.65%, compared to 1.34% without. The qualitative dependence on σ is similar to that in figure 1, but the values are smaller as real costs of both public and private capital are netted out.

Concluding remarks:

The long run effects of higher public investment affect income and well-being directly, and via induced changes in private behaviour, especially investment. Both channels need to be taken into account, and two parameters are critical. The elasticity of output with respect to public capital, and the elasticity of substitution between public and private capital. Effects are likely to be larger than suggested in OBR5, particularly when there is complementarity between the two types of capital. Effects will vary across different investment projects, requiring disaggregate analysis.

Figure 2: Consumption responses to a 27% increase in public capital with suboptimal levels of public and private capital



Blue line: no private investment response

Red line: with private investment response

[5] These seem large, although OBR5 para 4.12-4.27 suggest average private returns of 10.4%, which is 3 or 4 times larger than the Green Book's 3.5% or government's (marginal?) financing cost of 2.5%.