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A new UK overlapping generations model

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Abstract

This paper presents a new overlapping generations model of the UK economy, UK OLG. Built jointly by the Office for Budget Responsibility and HM Treasury, UK OLG analyses the long-term impact of shocks, trends and policies on the UK macroeconomy and public finances. The paper outlines the background to this model, before setting out UK OLG's structure. The model's general equilibrium framework captures interactions between households, the production sector and government. By explicitly modelling forward-looking behaviour in households both within and across generations, UK OLG creates robust simulations for the impact of shocks. The model also covers a simple treatment of bequests and inheritance. The paper then sets out how UK OLG has been calibrated to match recent UK economic data and reflect key features of the UK tax, spending and welfare systems. A detailed treatment of the age structure of the UK population allows the model to show the effects of shocks that affect age groups differently. Unanticipated shocks to earnings generate income and wealth inequality within generations, helping produce a realistic saving profile, including life-cycle and precautionary savings, and consumption smoothing. Finally, the paper demonstrates the model's properties with a series of sensitivity analyses to shocks.

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Executive summary

The remit of the Office for Budget Responsibility (OBR) requires it to investigate how economic and fiscal shocks, economic trends and policies affect the macroeconomy and public finances. The OBR employs a range of tools to make these assessments, including for the supply side and potential output. This paper introduces the UK Overlapping Generations model (UK OLG): a new, complementary tool developed jointly by teams at the OBR and HM Treasury.

Macroeconomists tend to model the macroeconomy either as an estimated system of aggregate equations, or as the result of behavioural decisions by a single, infinitely-lived 'representative' household or business. Overlapping generations (OLG) models keep the foundation of individual optimisation, but recognise the ways households change their behaviour over their life cycle, with retirement and finite lifespans explicitly considered. Government tax revenues reflect changing patterns of consumption and income over a lifetime, while components of government spending, such as state pensions, are intimately connected to demographic and economic characteristics. This makes OLG models useful for analysing the detailed impacts of fiscal policy on the macroeconomy.

UK OLG is an overlapping generations model for the analysis of the impact of shocks, trends and policies on the UK macroeconomy and public finances in the long term. The model is explicitly calibrated to the characteristics of the UK economy and demographic structure. Building on an established tradition in macroeconomics and at fiscal authorities, the UK OLG model is characterised by the following features:

- Like many standard theoretical macroeconomic models, it deploys a **general equilibrium framework** to capture the interactions between different parts of the economy, including households, the production sector and government.
- By modelling households' **forward-looking behaviour** explicitly, both within and across generations, the model can robustly estimate the impact of shocks and policies.
- A **detailed treatment of the age structure of the UK population** allows the model to simulate shocks that affect age groups differently and assess macroeconomic feedback effects.
- The model includes **unanticipated shocks to earnings** which generate income and wealth inequality within generations, helping to produce a realistic saving profile, including life-cycle and precautionary savings, and consumption smoothing.
- The model covers a simple treatment of **bequests and inheritance**. UK OLG has been calibrated to match recent UK data and reflect key features of the UK tax, spending and pension system.

- The model can operate as a **closed or open economy**, as appropriate for the analysis. The closed economy version allows interest rates to clear the market for assets, while the open economy has a fixed global interest rate, reflecting the UK's international economic position.

Illustrative scenarios presented in this paper showcase the model's ability to analyse the impacts of different tax policy, government spending and structural shocks. They highlight that the model is particularly well suited to analyse labour supply effects along the age dimension, and implications of household decisions on labour supply, saving and consumption over the life cycle for macroeconomic and fiscal aggregates. The OBR intends to use UK OLG to complement existing models for analysing long-term economic trends and policy impacts, and for building on our scenario capability. It will not replace the OBR macroeconomic model used to produce economy forecasts.

The model is sufficiently flexible to serve as the basis for future extensions and address specific policy and analytical questions. As such, it abstracts from many real-world complexities. Future extensions could model inequality in income and wealth within generations in more detail, add a richer production sector or provide more detail on inheritance.

1 Introduction

1.1 The remit of the Office for Budget Responsibility (OBR) is to "*investigate the impact of trends and policies on the public finances from a multitude of angles, including through forecasting, long-term projections and balance sheet analysis*" (HM Treasury, 2023). To assess the impact of these trends and policies on the public finances requires a thorough understanding of short-, medium- and long-run macroeconomic dynamics. The OBR employs a range of tools to make these assessments, including in its modelling of the drivers of potential output (OBR, 2022). This paper introduces a new, complementary tool developed jointly by teams at the OBR and HM Treasury: a dynamic stochastic overlapping generations general equilibrium model ('UK OLG'), calibrated to the UK economy and set up specifically to aid the assessment of the impact of shocks, trends, and policies on the economy and public finances in the long term.

1.2 Overlapping generations (OLG) models have long been used by fiscal policy institutions like the Congressional Budget Office (CBO) and the Joint Committee on Taxation in the United States, and the European Commission (see below for references). OLG models are particularly useful for fiscal policy analysis because the presence of multiple generations within the model allows for a more realistic representation of household behaviour. The choices of one generation are affected by the historical decisions of now-older generations when they were in the same position. For example, if all households save intensively for retirement, the larger stock of savings would lower the interest rate in a closed economy. Younger generations would then have to save even more to achieve the same level of retirement assets. In this situation, slightly higher consumption would benefit all generations, but is not individually optimal given the constraints arising from previous generations' decisions. OLG models can capture these types of interactions and behaviours across generations, and so allow for a richer analysis of policy, where government debt, detailed tax design and redistribution all have meaningful macroeconomic impacts (Weil, 2008).

1.3 UK OLG contains a number of features filling gaps in the OBR's current long-term modelling toolkit:

1. a **general equilibrium framework** in which interactions between fiscal policy, households, firms and macroeconomic aggregates like income and long-run interest rates are formally modelled and reconciled;

2. households' forward-looking behaviour is explicitly modelled (**micro foundations**) so we can assess the impact of shocks and policies without assuming that past aggregate relationships will always stay the same despite a change to individuals' economic incentives (addressing the Lucas critique);

3 the interactions between multiple generations that vary in age, income and wealth is also explicitly modelled, allowing for insights beyond aggregate macroeconomic effects, in particular regarding policies or shocks that affect only a fraction of the population (**heterogenous agents**);

4 considering unanticipated earning shocks (**stochastics**) generates precautionary saving and enables us to match data on saving behaviour, as well as giving a richer understanding of how tax and spending impacts vary due to the different effects on households at different levels of income and wealth;

5 calibration to UK data enables us to assess **UK-specific** dynamics and impacts relative to other OLG policy models;

6 by accounting for the fiscally most important features of the economy while remaining analytically tractable, and by making use of a flexible solution method (the Value Function Iteration Toolkit developed by Kirkby, 2017, 2023a), the present framework can serve as a **core model** for analysis and a basis for future extensions.

1.4 OLG models follow a long-running tradition dating back over 60 years. It originates in Samuelson's (1958) seminal work explicitly modelling a succession of generations in place of an infinitely lived representative agent. Blanchard (1985) extends and simplifies this framework using Yaari's (1965) assumption of probabilistic (rather than deterministic) death to analyse long-run effects of fiscal policy. Summers (1981) pioneered the use of OLG to analyse tax reforms. The basis for current OLG policy models was provided by Auerbach and Kotlikoff's seminal 1987 book, which added a number of innovations that were crucial for policy analysis. These include the assumption of perfect foresight (rational expectations) and endogenous labour supply which responds to the wage profile over age. Of the many extensions of the Auerbach and Kotlikoff (1987) framework, Fullerton and Rogers (1993) consider heterogeneity within generations and across multiple production sectors and Engen and Gale (1993l) model earnings uncertainty which gives rise to precautionary savings.[4]

1.5 OLG models currently in use by official fiscal forecasting institutions build off the Auerbach and Kotlikoff (1987) framework.

- The US CBO uses versions of an OLG model characterised by multiple skill levels within each age group and policy-based transition paths (Nishiyama, 2003, 2015).
- The OLG-Computable General Equilibrium model employed by the US Joint Committee on Taxation (Diamond and Zodrow, 2013) considers transition dynamics, multiple income groups and multiple production sectors.
- The European Commission Joint Research Centre's EDGE-M3 OLG model (d'Andria et al., 2020) also explicitly models multiple earning-ability types per generation,

[4] See Diamond and Zodrow (2013) for a review of the OLG literature with a focus on fiscal policy analysis.

computes transition paths and uses rich microeconomic data to calibrate European tax, benefits and earnings characteristics.

- The Australian Treasury's OLG model OLGA (Cai et al., 2023) accounts for intragenerational heterogeneity, input-output linkages across a number of production sectors, transition paths and Australia's trade and financial linkages with the rest of the world.

- The FISK OLG model of the Austrian Fiscal Council (Schuster, 2021) captures intricacies of the Austrian pension system and is solved for non-steady state equilibria such that non-stationary effects of long-lasting reforms can be assessed.

1.6 The UK OLG model introduced in this paper incorporates the main features of these policy OLG models. It calibrates to the UK economy, tax, benefit and pension systems, which enables users to analyse UK-specific shocks, policies and long-run macroeconomic effects. The model solves for a long-run equilibrium and without taking a view on the transition dynamics. We also expand on previous policy models by incorporating stochastic earning shocks which in our model generate intra-generational heterogeneity.

1.7 This paper proceeds as follows. Section 2 sets out the structure of the UK OLG model. Section 3 outlines the solution algorithm. Section 4 explains how we calibrated the model. Section 5 illustrates the simulation properties of the model using different supply-side and policy scenarios. Section 6 concludes.

2 Model structure

2.1 This section sets out the model equations and definition of equilibrium.[5]

2.2 Households are heterogeneous by age and by individual income shocks. This generates inequality in income and assets both between and within generations. They solve an intertemporal consumption and hours worked decision, conditional on their income shocks, eligibility for welfare transfers and their probability of survival to older ages. The very oldest households also gain direct utility from leaving bequests, and all bequests are distributed equally across the previous generation of households (aged 55 to 75). Any assets left on death above the optimal level of bequest, or left behind by those dying younger, are treated as an 'accidental' bequest. Household decisions are subject to a budget constraint depending on their income, assets and bequests. Asset holdings may not be negative.

2.3 The government taxes income and consumption and makes age-dependent welfare payments, with other taxes and spending covered by a residual balance term. The model is solved by the government keeping debt at a fixed share of GDP and the government covers deficits by issuing bonds, which enter the stock of assets. As described below, the scale of the adjustment in residual tax and spending required to ensure debt remains at a fixed share of GDP shows the fiscal impact, and plausibility, of a given scenario in the model. Total pensions are covered by a combination of households' private savings and the government's age-dependent welfare payments (largely reflecting state pensions). Government policy is modelled as of March 2024, in order to be consistent with that period's OBR *Economic and fiscal outlook* (*EFO*) forecasts.

2.4 Production happens in a representative firm in a competitive market, with no randomness in overall output (there is no aggregate uncertainty). Non-bond assets are used as capital for production, and the entire stock of assets is distributed across households, who earn interest on their holdings. While this is a relatively simple production sector, the OBR draws on computable general equilibrium models that cover this part of the economy in more depth.

2.5 Markets clear under four conditions. First, the interest rate on assets must equal one of two values. If the economy is closed to international financial markets, the interest rate equals the (fully deterministic) marginal product of capital. If the economy is fully open, the interest rate is set at global levels. Second, wages equal the marginal product of labour. Third, government activity is fully funded by tax revenues and bond issuance. Fourth, government debt equals its user-defined target by adjusting a residual budgetary item. The economy reaches equilibrium when for a given set of prices and government policies, consumption, hours, assets and bequests solve the household's lifetime optimisation problem.

[5] The notation largely follows Kirkby (2023b).

Households

2.6 **Households** are the smallest unit of account in the model. They are heterogeneous with respect to their age j, age-dependent average productivity κ^j, by-household stochastic shocks to labour income each period and, as a result, beginning-of-year wealth a. We abstract from characteristics like gender, marital status, and the number of children. Households become economically active at age 20 and then live up to J=100 years. Death is stochastic such that households base decisions on an age-dependent probability of survival s^j each year. By construction of the grids in the model, asset holdings cannot be negative, implying households cannot be in net debt in equilibrium.

2.7 To model the **demographic profile of the population** we denote the number of economically active households of each age μ^j, with j running from age 20 to 100. We further assume that the overall population grows at an annual rate of n such that a cohort of age j is $(1+n)$ times the size of cohort $j+1$, before accounting for death. For tractability we normalise the total population to one such that μ^j corresponds to the share of age group j in the economically active population:

$$\mu^{j+1} = \frac{1}{\sum_{k=20}^{J} \mu^k} \cdot \frac{s^j \mu^j}{1+n} \quad \text{for} \quad j = 20, 21, \ldots, J \tag{1}$$

2.8 Each year t, every individual household receives an earnings shock. These shocks to households' hourly earnings come in two types. One is a slowly-decaying (stationary, first-order autoregressive, or AR(1)) component z to capture real-world persistence in earning differentials within each generation. The second is a transitory (independent and identically normally distributed, or i.i.d.) component e to reflect annual variation in pay. Each household optimises their decisions about consumption c and time spent in work h over their whole expected lifetime, conditional on earnings shocks and their wish to leave behind bequests. In the following individual household decision and constraint equations, for simplicity we omit the household subscript of i. Written as a value function, each individual household's **life-cycle problem** is:

$$V_t(a_t, z_t, e_t, j) = \max_{c_t, h_t, a_{t+1}} \left\{ \frac{\left[c_t^{\sigma_1}(h^{max} - h_t)^{1-\sigma_1}\right]^{1-\sigma_2}}{1-\sigma_2} + I^j \beta\left(1 - s^j\right) warmglow(a_{t+1}) \right.$$
$$\left. + \beta s^j E_{z_{t+1}, e_{t+1}}[V_{t+1}(a_{t+1}, z_{t+1}, e_{t+1}, j) | z_t] \right\} \tag{2}$$

subject to the following budget constraint:

$$(1+\tau^c)c_t + a_{t+1} = NetIncome_t + (1+r)a_t + (1+r) \cdot \Omega^j \cdot \sum Bequests_{t-1} \tag{3}$$

where β is the discount rate. I^j is an indicator function that is set to $I^j = 1$ if $j \geq j^*$, and $I^j = 0$ otherwise, for a calibrated age j^* above which households earn contemporaneous utility from leaving wealth behind for future generations, captured by the $warmglow$ term.[6] Ω^j is an indicator for whether a household is in the generation receiving bequests (55-75 year-olds) from those who die in the previous period. r is the real interest rate, $NetIncome_t$ is income from labour and government transfers net of income tax and national insurance contributions, and τ^c is the rate of value added tax. Households have rational expectations about future bequests left to them. Terms of the value function and budget constraint are defined in more detail below.

2.9 The **utility function** in (2) takes the form of a Cobb-Douglas function with constant relative risk aversion (CRRA). σ_1 measures the share of consumption in instantaneous utility and σ_2 measures relative risk aversion. As consumption may vary over time, this expected variation is a form of risk, so a more risk-averse household will aim for a smoother path of consumption over time. This means risk aversion implicitly determines a household's time preference over when to consume. The function is non-separable such that the marginal utility of leisure is increasing in consumption. This is consistent with a balanced growth path (hours worked are stable in steady state despite the presence of productivity growth which we introduce below). With the functional form above, labour supply not only depends on after-tax wage rates but also on household wealth. The Congressional Budget Office (CBO) model (Nishiyama, 2013) and Australian OLGA model (Cai et al., 2023) use the same functional form.

2.10 The elasticity of hours in response to one-time temporary changes in after-tax labour income (**Frisch elasticity**) is:

$$FE = \frac{h^{max} - h}{h} \cdot \frac{1 - \sigma_1(1 - \sigma_2)}{\sigma_2} \tag{4}$$

where h^{max} is the maximum time households have available to spend at work.

2.11 **Net income** is composed of labour income and age-dependent government transfers, net of income tax and employee national insurance contributions:

$$NetIncome_t = w_t \kappa^j e^{z_t + e_t} h_t - IncomeTax_t - EENIC_t + TRAN_t^j \tag{5}$$

w_t is the average hourly wage rate, which is an aggregate variable across the whole economy. κ^j is a parameter related to age-specific productivity that shifts hourly wages up or down depending on age, and is the same across individuals of a given age. z_t is the persistent AR(1) shock to earnings and e_t the temporary i.i.d. shock, and both apply at household level (we take the by-household i subscript as implicit).

$$z_{t+1} = \rho^z z_t + \epsilon_t^z, \;\; \epsilon_t^z \sim N\left(0, {\sigma_{\epsilon,z}}^2\right) \tag{6}$$

[6] Through forward expectations the warm glow bequest motive is taken into account by household from the beginning of their economically active life. The description of 'warm glow' is used in the literature to distinguish this type of bequest altruism from others and is not intended as a real-world description (see for instance Acemoglu (2009), Chapter 9).

$$e_t \sim N(0, \sigma_e{}^2) \tag{7}$$

Income tax payments $IncomeTax_t$, national insurance contributions paid by employees $EENIC_t$ and age-dependent government transfers $TRAN_t^j$ including state pension and other income-replacing welfare benefits, are defined below (from paragraph 2.22).

2.12 The **decision to retire**, i.e., to provide zero or close to zero hours of work, is endogenous and depends on the combination of age-dependent productivity κ^j, age-dependent transfers including the state pension, and tax payments.[7]

2.13 When households die, their assets are distributed to surviving households as **bequests** in the next period. We assume that the total sum of bequests are redistributed equally to the next surviving generation, i.e., those aged 55 to 75, in the form of lump-sum payments.[8]

2.14 In the real world, households have a range of reasons for leaving bequests. These include precautionary motives as retired households face the risk of rising health and care expenses, the desire to leave bequests to following generations, or inability to decumulate wealth held in housing (French et al., 2023). There are many complex ways one could account for this within an OLG model. We follow the Australian OLGA model (Cai et al., 2023), the US Joint Committee on Taxation model (Diamond and Zodrow, 2013) and the European Commission model (d'Andria et al., 2020) and model an ad hoc **'warm glow' utility** of leaving bequests behind after death:

$$warmglow(a) = \phi_1 \frac{(a - \phi_2)^{1-\sigma_2}}{1 - \sigma_2} \tag{8}$$

where ϕ_1 captures the importance of the 'warm glow' motive relative to consumption and leisure in the utility function. ϕ_2 is the end-of-life target for assets to be bequeathed. And σ_2 is the CRRA risk aversion/time preference parameter defined above. The motive to leave bequests behind kicks in from age j^*, i.e., indicator I in equation (2) is set to 1 if age $j \geq j^*$. We use j^* to calibrate old-age assets. While this means that the warm glow motive features directly and contemporaneously in households' utility function from age j^*, households plan for these bequests, and in expectation receive anticipatory utility, throughout their life through the forward-looking nature of utility.

2.15 With the time of death stochastic, **accidental bequests** occur when households pass away before reaching the age j^* and leave more wealth behind than would be optimal if the timing of death was certain. We denote the total assets of age group j at time t a_t^j.

$$\Omega^j \cdot \sum Bequests_{t-1} = AccBequestLeft_{t-1} + OptimalBequestLeft_{t-1} = \frac{\sum_{j=20}^{J} \mu^j a'^j_t (1 - s^j)}{1 + n} \tag{9}$$

[7] For calibration purposes, and to rule out households returning to work to pay for bequests, we force retirement, i.e., $h_t = 0$, at age $j = 80$, well above the state pension age.

[8] We abstract from a more realistic but complex distribution of bequests as well as inheritance tax. These model features can be considered in future extensions.

Firms

2.16 The **production side** of the model is simple and consists of a representative firm that operates in a perfectly competitive environment. We also assume there is no aggregate uncertainty, meaning in equilibrium there are no business cycles and no random shocks to the overall economy. Output is produced using a constant-returns-to-scale Cobb-Douglas production function.

$$Y_t = AK_t^{\alpha}((1+g)^t L_t)^{1-\alpha} \quad (10)$$

where A is the aggregate level of productivity. K_t is the aggregate stock of private sector capital employed by firms. In the closed economy version of the model the capital employed by firms equals the total capital provided by households. In the open economy version, it will also include net foreign assets. See paragraphs 2.39 and 2.40 for more discussion of the equilibrium condition for capital markets. α is the capital share, g is the growth rate of labour-augmenting technology such that output per person grows at rate $(1+g)^t$ which is the sum of total factor productivity growth and capital deepening. Capital depreciates each year at rate δ. This approach follows the standard in the literature and is consistent with the OBR's framework for the analysis of potential output (OBR, 2022). Aggregate labour, adjusted for individual productivity is given by:

$$L_t = \sum_{j=20}^{J} \mu^j \kappa^j e^{z_t+e_t} h_t \quad (11)$$

2.17 When we refer to modelled GDP throughout this report, we are implicitly referring to total market production. This excludes public sector production, which we leave unmodelled except for its fiscal cost entering the residual budget item (see below for more detail).

2.18 The model can be run with the assumption of a small **open economy** such that r is exogenously determined on global financial markets and excess demand or supply of assets is implicitly met through foreign investment. Depending on the application of the model this can be changed to an alternative specification for a **closed economy** in which r is determined endogenously.

2.19 The model does not explicitly incorporate **housing assets** in its structure. Residential property is an asset that differs in many ways from the financial assets considered here, including by being more illiquid than (non-pension) financial savings and by providing housing services during the duration of asset build-up. It therefore warrants a separate treatment in the model which can be considered in future extensions.

Pension system

2.20 The **UK pension** system is a two-tiered system where individuals that retire receive a state pension and draw on savings in private (including workplace) pension pots. The state pension is a form of welfare transfer and explained below (from paragraph 2.22). Private pensions are accumulated in defined benefit or, increasingly, defined contribution pension pots, most of which are funded by pension fund managers investing pension contributions in bonds (here government bonds) and equity (here private capital). In addition to pensions, households may hold separate savings in bonds and equity which they can use to smooth consumption and leisure over their lifetime and support their standard of living in old age. To keep the model tractable while capturing the features most important to questions of pension policy, we therefore consider total household (non-residential) assets as the sum of government debt and private capital. We interpret a share of these assets as private pensions, while the remainder is interpreted as other private savings. For simplicity we abstract from annuities; instead, households draw on total assets a, i.e., private pensions and other savings, to finance consumption during retirement.

2.21 Given the preferential treatment of **pension contributions** in the tax system, we define employee and employer pension contributions relative to earnings, respectively, as follows:

$$eeppc_t = \varphi^{ee} w_t \kappa^j e^{z_t + e_t} h_t \tag{12}$$

$$erppc_t = \varphi^{er} w_t \kappa^j e^{z_t + e_t} h_t \tag{13}$$

where φ^{ee} and φ^{er} are average contribution rates (see Table 4.1). The structure of the model does not specifically reserve these contributions into savings, but simply treats them as an extra portion of labour income that is exempt from tax. The contribution rates are exogenous, and not subject to household choice. By treating pension relief effectively as an extra tax allowance on labour incomes, this makes more funds available to save for retirement. However, contrary to real-world outcomes, it is possible for households to choose not to save the entire pension relief amount.

Government

2.22 The government taxes income, charges national insurance contributions, taxes consumption and collects residual tax to raise revenue. On the expenditure side, it pays for age-dependent welfare transfers, debt interest payments and remaining government consumption and investment. We allow the government to run a budget deficit or surplus in equilibrium by issuing or repaying government bonds. To solve this version of the model, we require the government to follow a budget rule that holds debt relative to GDP stable. In future iterations of the model, we plan to explore using different government budget rules to solve the model. To achieve stable debt as a share of GDP, the government adjusts tax and spending via the residual budgetary item (described below). The scale of the change in this variable is a measure of the fiscal adjustment required to keep the debt-to-GDP ratio stable, and allows the user to determine if such an adjustment would be feasible or if the model is indicating that the fiscal position is unsustainable.

2.23 To model the complex non-linearities of the progressive **income tax system**, the literature tends to interpolate the relationship between marginal tax rates and incomes using non-linear functions (see the European Commission model in d'Andria et al., 2020, or the US

CBO model in Nishiyama, 2013). However, marginal income tax rates in the UK tend to follow fairly simple thresholds. We therefore follow the approach of the Australian OLGA model (Cai et al., 2023) and model income tax bands explicitly.

2.24 The tax base for each individual household's income tax is made up of labour income and taxable benefits, including state pensions. Income tax is also levied on withdrawals from private pension funds. We capture pension withdrawals by accounting in the tax base for the difference in assets between two periods $a_t - a_{t+1}$. Only if it is positive, i.e., the next-period asset stock is smaller than current-period assets, is it subject to income tax. In addition, not all withdrawals from private assets represent a pension payment. We therefore approximate pension payments by modelling the tax-relevant share of total household financial asset withdrawals as the share of assets held in pensions $pshare$.[9] The below once again omits the household subscript i.

$$IncomeTaxBase_t = w_t \kappa^j e^{z_t + e_t} h_t + TRAN_t^j + pshare \cdot \max(0, (1+r)a_t - a_{t+1}) - ppcrelief_t \quad (14)$$

Tax relief is granted on employee contributions to private pensions up to an annual allowance (AA).

$$ppcrelief_t = \begin{cases} eeppc_t & \text{if} \quad eeppc_t < AA \\ AA & \text{otherwise} \end{cases} \quad (15)$$

2.25 There are three marginal income tax thresholds TH^1, TH^2, TH^3 above which three marginal tax rates apply τ^1, τ^2, τ^3. Income tax payable then depends on the tax bands in which the $IncomeTaxBase_t$ falls, and the three marginal rates of income tax. Income below threshold TH^1 is not taxed. Additional income that falls between thresholds TH^1 and TH^2 is subject to the basic rate of income tax τ^1. Anyone with a tax base between thresholds TH^2 and TH^3 pays the basic rate on the fraction of income that falls between thresholds TH^1 and TH^2 and the higher rate τ^2 on the fraction of the tax base that lies above threshold TH^2. Similarly, if the income tax base exceeds threshold TH^3, all three marginal rates are applied to respective fractions of income.

$$IncomeTax_t = \begin{cases} \tau^1(IncomeTaxBase_t - TH^1) & \text{if} \quad TH^1 < IncomeTaxBase_t \leq TH^2 \\ \tau^1(TH^2 - TH^1) + \tau^2(IncomeTaxBase_t - TH^2) & \text{if} \quad TH^2 < IncomeTaxBase_t \leq TH^3 \\ \tau^1(TH^2 - TH^1) + \tau^2(TH^3 - TH^2) + \tau^3(IncomeTaxBase_t - TH^3) & \text{if} \quad TH^3 < IncomeTaxBase_t \\ 0 & \text{otherwise} \end{cases} \quad (16)$$

[9] We abstract from capital gains tax that would be due on withdrawals of non-pension financial savings.

2.26 We specify **National Insurance contributions** (NICs) similar to income tax, though we only model the contributions of employees explicitly. Employees are exempt from NICs for the contributions their employers make to pension funds, but not for their own contributions. The base for employee NICs is made up only of labour income:

$$NICBase_t = w_t \kappa^j e^{z_t+e_t} h_t - erppc_t \tag{17}$$

2.27 There are two thresholds $NICTH^1$, $NICTH^2$ for employees,[10] and total employee NICs are:

$$EENIC_t = \begin{cases} eenr^1(NICBase_t - NICTH^1) & \text{if} \quad NICTH^1 < NICBase_t \leq NICTH^2 \\ eenr^1(NICTH^2 - NICTH^1) + eenr^2(NICBase_t - NICTH^2) & \text{if} \quad NICTH^2 < NICBase_t \leq NICTH^3 \\ 0 & \text{otherwise} \end{cases} \tag{18}$$

where $eenr^1$ and $eenr^2$ are marginal tax rates.

2.28 The government also taxes consumption. Revenue from **value added tax** (VAT) is:

$$VAT_t = \tau^c c_t \tag{19}$$

where τ^c is the effective rate of VAT.

2.29 In reality, the government levies a range of other taxes and levies on households and firms. We do not model these explicitly, though further extensions of the model could split them out. Instead, we consider the remaining aggregate tax revenue per person T_t^0 in a residual budgetary item explained below.

2.30 On the expenditure side, we model age-dependent **welfare transfers** per person, $TRAN_t^j$. These capture mainly the state pension which every household receives after reaching retirement age independent of previous earnings. The transfer variable also accounts for other welfare transfers, like Universal Credit, and other pension credits and benefit payments, but abstracts from means testing, i.e., conditionality on other income. Note that all welfare transfers, including state pension, are paid out of general taxation rather than separate budgets. We model age-dependent transfers as a function of average welfare transfers and age-specific scaling parameters γ^j.

$$TRAN_t^j = \gamma^j TRAN_t^0 \tag{20}$$

where $TRAN_t^0$ captures average welfare transfers. We use it to calibrate total transfer payments relative to GDP.

[10] We make further simplifications and abstract from different categories focusing on employees only.

2.31 Other than welfare transfers, government expenditure includes a range of government consumption and investment items. We collect **remaining non-interest government expenditure** per person in a residual spending category G_t^0. G_t^0 is assumed to be age-independent and does not yield any economic benefit in the model (it is not part of households' utility function).[11]

2.32 Instead of explicitly modelling residual tax and spending, T_t^0 and G_t^0, we consider a **residual budgetary item** (RBI_t) that implicitly collects both and equals net residual spending. This is because neither residual tax nor spending features in the household utility function. We only model market production, so public goods and tax on non-modelled activity are treated as an exogenous factor. They do not enter household utility functions or the production decisions of businesses. The residual budgetary item is a measure of the fiscal gap relative to the budget rule defined below (the required adjustment to prevent debt increasing).

$$RBI_t = G_t^0 - T_t^0 \tag{21}$$

2.33 The government finances any difference between revenue and expenditures by issuing **government debt**, B_t. Government debt provides an asset for households, as well as foreign investors, to invest in, alongside private capital. We assume government debt pays the same return r as private capital.[12] **Debt interest payments** are therefore rB_t.

2.34 To close the model, we need to define a path for government debt. We define the following **budget rule** whereby the government holds government debt stable as a share of GDP. In the literature, different instruments are used to achieve fiscal targets, including endogenous changes in income tax rates, welfare transfers or non-welfare spending. We let the residual budgetary item RBI_t adjust to meet the debt-to-GDP target.[13] Given RBI_t does not feature in households' utility, this means any tax reductions or welfare increases do not generate offsetting effects on economic benefits to households. However, it does not mean that government deficits are costless to the government. Instead, the assumption enables us to quantify the long-run impact of different policies on the government's financing needs. While the government in reality will be required to change tax or spending to meet the budget rule, the assumption allows us to remain agnostic about the choice of instrument. Closing the model through non-distortionary residual net spending also enables us to estimate the direct economic impact of shocks and policies, unaltered by the indirect effect offsetting tax or spending will have.[14]

[11] Future extensions could consider age-dependent components, like spending on health, adult social care and education, and allow for feedback effects on the economy, like effects of government investment on the whole-economy capital stock.
[12] This way we avoid having to model an ad hoc preference to hold government debt versus private capital in the absence of aggregate uncertainty.
[13] Other specifications are possible and can be experimented with in future extensions.
[14] The debt-to-GDP target can be changed in simulation. In the closed economy setting equilibrium interest rates r adjust to the stock of government debt. Simulations can also be designed with explicit assumptions about debt, tax or spending to achieve a given level of RBI_t.

Market clearing

2.35 In steady state, labour productivity is assumed to grow at rate g and the population grows at rate n. To solve the model, we need to **turn model variables into stationary variables**, accounting for the fact that different endogenous variables grow at different rates. For each model variable x_t, we denote the stationary equivalent using 'hats' $\hat{x}_t$. For example, for a variable x_t that in steady state grows both in line with population and productivity growth, the stationary equivalent is $\hat{x}_t = \frac{x_t}{(1+n)(1+g)}$.

- Variables that grow at rate g include w_t, per person fiscal variables, and c_t.
- Variables that grow at rate n include L_t.
- Variables that grow both at g and n include aggregate variables like Y_t and K_t.
- Inherently stationary variables include r.

2.36 We define the **market clearing conditions** under the assumption that the labour market, international capital market and goods market are perfectly competitive. This provides the first set of equations that define the stationary equilibrium of the model.

2.37 Labour markets clear such that wages equal the marginal product of labour:

$$\hat{w} = (1 - \alpha)A\hat{K}^{\alpha}\hat{L}^{-\alpha} \tag{22}$$

2.38 The total amount of capital employed by firms satisfies the condition that the interest rate equals the marginal product of capital less depreciation:

$$r = \alpha A\hat{K}^{\alpha-1}\hat{L}^{1-\alpha} - \delta \tag{23}$$

2.39 The capital market clearing condition depends on whether the model is in its open or closed economy setting. In the closed economy version, the firm's capital asset holdings K_t equal total household assets minus the stock of government debt:

$$K_t = \sum_{k=20}^{J} \mu^k a_t^k - B_t \tag{24a}$$

In the closed economy, real interest rates from equation 23 adjust endogenously to satisfy this condition. This equilibrium interest rate is fully deterministic, as there is no aggregate uncertainty in the model.

2.40 In the open economy setting, the real interest rate given in equation 23 is fixed to the prevailing global rate. That means the total capital employed by firms may not coincide with total capital provided by households. The difference is then made up by the net foreign assets position NFA_t:

$$K_t + NFA_t = \sum_{k=20}^{J} \mu^k a_t^k - B_t \quad (24b)$$

If NFA_t is positive, households provide more capital than domestic firms employ, so households use the excess capital to buy foreign assets. If NFA_t is negative, then the total capital employed by firms exceeds the total capital provided by households, and the difference is made up by foreign assets coming into the country. This pushes down the net foreign assets position because overseas asset holders now own more of the economy's productive resources.

2.41 Government expenditure, revenue and changes in the stock of government debt balance. Note that debt enters through interest payments, the stock of debt and its growth between two time periods, giving rise to a complex growth rate interaction.

$$\widehat{TRAN} + \widehat{RBI} + (r - n - g - ng)\hat{B} = \widehat{IncomeTax} + \widehat{EENIC} + \widehat{VAT} \quad (25)$$

2.42 The government achieves a fixed target debt stock to GDP ratio set by the user. In this paper, we set the target at 100 per cent as a simple value close to the current UK figure.

$$\frac{\hat{B}}{\hat{Y}} = 1 \quad (26)$$

2.43 Bequests left equal bequests distributed to inheriting generations:

$$\widehat{Bequest} = \frac{\widehat{BequestLeft}}{(1+g)(1+n)} \quad (27)$$

2.44 A stationary equilibrium for variables r, $\hat{w}$, tax and spending is further defined by the following **equilibrium conditions**.

- Given prices and government policies, the household value function V and the related bequests schedule solve the household problem.

- Aggregate variables are based on household schedules, government policies and the household distribution.

3 Solution algorithm

3.1 There is no unique, commonly used solution algorithm for complex overlapping generations (OLG) policy models like UK OLG. Instead, tailor-made approaches tend to be used that are specific to each model, including by policy institutions (d'Andria et al., 2020; Cai et al., 2023; Schuster, 2021).

3.2 To solve the model, we employ the Value Function Iteration (VFI) toolkit, developed and made available by Robert Kirkby (Kirkby 2017, 2023a, see bibliography for web link). The toolkit is the closest to a universal solution method for complex economic models like OLG that is currently available. It is implemented in MATLAB and enables users to solve non-linear economic models by value function iteration using Graphics Processing Unit (GPU) and Central Processing Unit (CPU) parallelisation. Employing value function iteration (a form of 'dynamic programming') to solve the model, rather than solving it analytically, provides us with substantial flexibility to adapt the model in the future without having to alter the solution method. This is because it is based on full discretisation of the state space – different possible solutions are discretely modelled, and a loss function determines which solution is picked. This tends to make value function iteration a slow solution method. However, by employing parallelisation, the toolkit enables us to solve the model reasonably quickly. Once fully calibrated with initial values set close to model solutions, the core model solves in less than 2 minutes, and a simple scenario making a small change can take less than 5 minutes. Flexibility is further aided by the very general set-up and robustness of the VFI toolkit. A disadvantage is that the solution algorithm itself is not defined explicitly in our code, though it is well documented and peer reviewed.

3.3 To solve the OLG model described in the previous section, we use the VFI toolkit as follows. We first set parameters and import external data, also stored as parameters. This includes setting the initial distribution of households at birth and households' age distribution. We then specify grids for discretisation; these also provide bounds on certain model outputs. We further set initial values for general equilibrium variables, generally those that correspond to model outputs from the main calibration. The main input into the toolkit is the return function, which corresponds to the utility function (equation 2), its constraints and the discount rate. Finally, equilibrium conditions are defined and we solve for general equilibrium. This requires defining aggregate variables and market clearing conditions as well as the government budget rule (see previous section). We also use the approach applied to general equilibrium variables to calibrate total welfare payments as a share of output, the bequest motive parameter ϕ_2, and a switch parameter to convert model units into £ thousands.

4 Calibration

4.1 We calibrate the model such that for a baseline, model outputs resemble the UK economy as closely as possible. We focus on demographics, the life-cycle distribution of earnings and wealth as well as main fiscal aggregates. Behavioural parameters are taken from the literature spanning different advanced economies and, where required, are adjusted to improve calibration for the UK. Where variables are calibrated to OBR forecasts, we use the March 2024 *Economic and fiscal outlook* (*EFO*).

4.2 Units in the model are adjusted for both population and productivity, making them hard to interpret, so we use a scaling parameter S to translate between real world data and model variables. The shift parameter is calibrated such that real GDP per person in the model is equal to its OBR forecast value of £35,100 in fiscal year 2028-29.

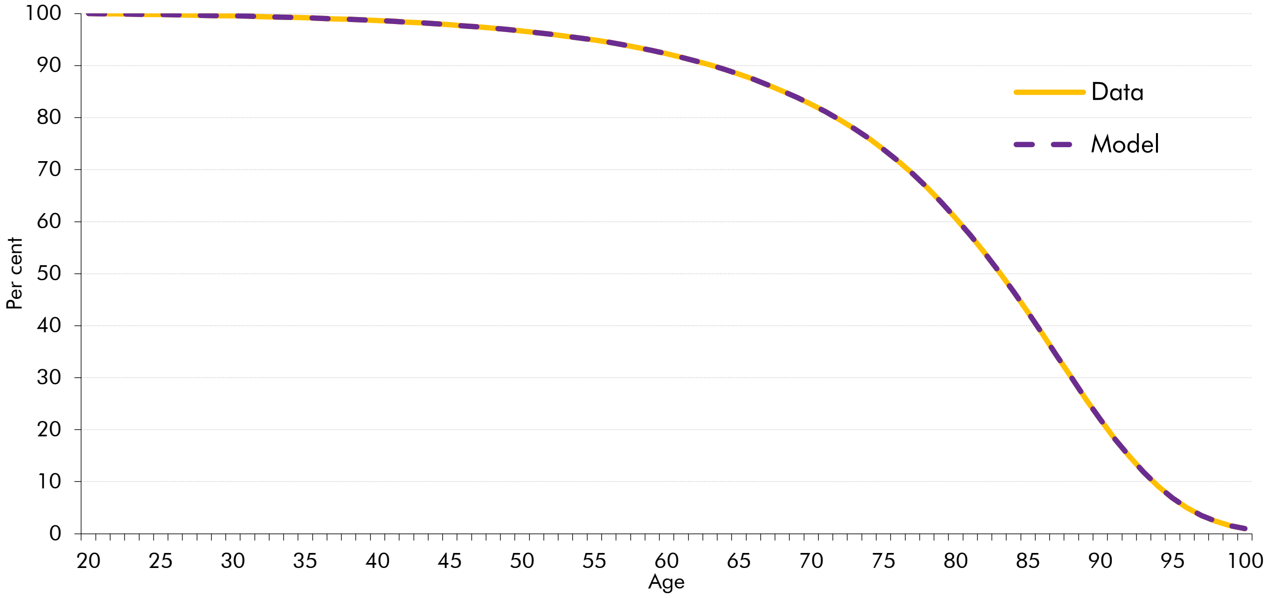
4.3 Tables 4.1 and 4.2 at the end of this section report the main model parameters we chose and a summary of model outputs relative to their real-world equivalents. The remainder of this section explains in detail the areas the calibration focused on.

4.4 Where relevant, we present baseline figures for both an open economy calibration with global real interest rates set at 5 per cent and a closed economy calibration where the equilibrium interest rate is endogenous. The interest rate set in the open economy solution (close to measures of the weighted average cost of capital) is lower than the equilibrium rate in the closed economy setting. This prompts foreign capital inflows, and a higher capital-to-labour ratio. This would normally imply a higher average wage, but since we are holding output per person constant, a higher capital stock necessarily implies lower labour productivity. In this baseline, just over 43¾ per cent of capital is foreign-owned. This is slightly higher than OBR forecasts, which project foreign claims on UK equity equalling just over 40½ per cent of the capital stock's value in 2028-29.

Demographics

4.5 We use mortality data from the Office for National Statistics (ONS) to recreate the age profile of the UK population within UK OLG. By calculating population share parameters μ^j from ONS death rates, the model-generated probability of reaching a certain age matches the actual age distribution exactly (Chart 4.1).

Chart 4.1: Probability of reaching each age



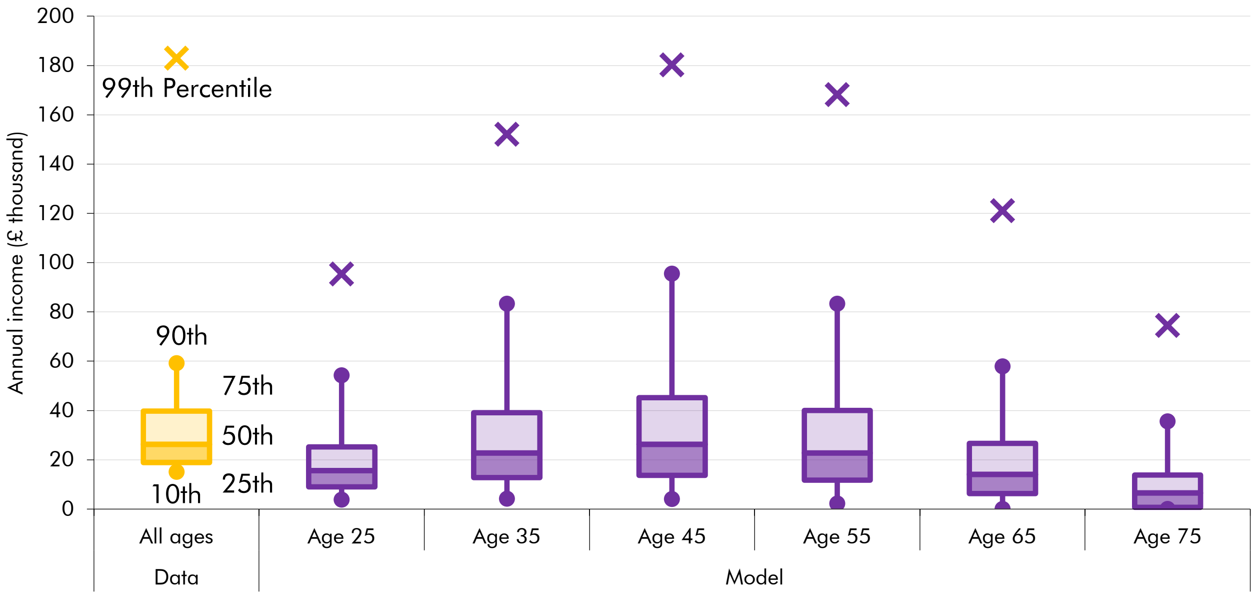
Source: ONS, OBR

Income and wealth

4.6 We use data from the ONS Annual Survey of Hours and Earnings (ASHE) on average earnings by age to calibrate age-specific productivity parameters κ^j. These κ^j are fed directly into the model as a data import. They determine labour supply decisions, income and wealth.

4.7 To benchmark the intra-generational distribution of earnings, we employ data from HM Revenue and Customs (HMRC). This data is only available for the working population as a whole rather than split by age group. The intra-generational distribution of earnings is determined by the persistence and standard deviation of the earnings shock z_t (calibrated to $\rho^z = 0.9$ and ${\sigma_{\epsilon,z}}^2 = 0.05$, respectively) and the standard deviation of the i.i.d. shock e_t (calibrated to ${\sigma_e}^2 = 0.7$). Following Kirkby (2023b), we use estimates in Fella et al. (2019) as the starting point, simplify by making ρ^z independent of age, and calibrate to match the data. To replicate the fat tail of the income distribution at high income levels, we set both the persistence parameter ρ^z and the standard deviation of the i.i.d. shock ${\sigma_e}^2$ to higher values than the literature. Due to the persistence of shocks, the intra-generational earnings distribution is further determined by initial values for z_t and e_t (at age 20). Chart 4.2 shows that with this parameter setting we match median earnings and earnings at the top of the distribution reasonably well. Earnings of households in lower percentiles are somewhat underestimated.

Chart 4.2: Intra-generational earnings distribution (closed economy)

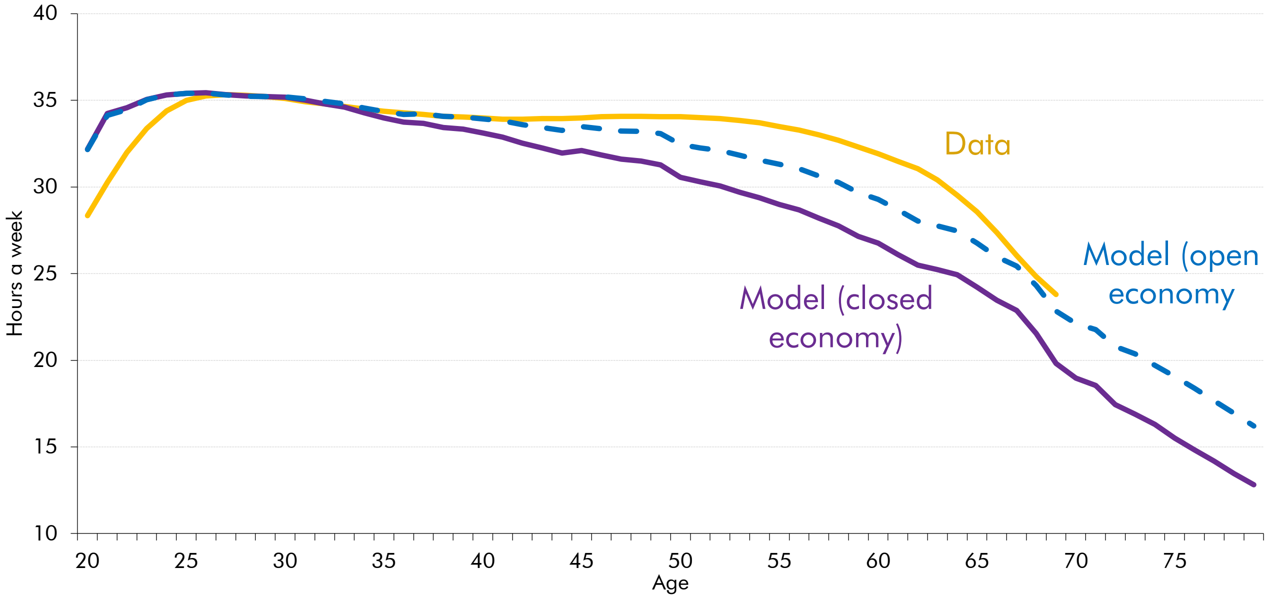


Source: ONS, OBR

4.8 The distribution of labour income over age is determined by households' labour supply decisions. Labour supply decisions in turn depend on age-specific productivity profiles, the stochastic earnings shock, and incentives to leave assets behind for old age and bequests (the warm glow motive). Like for labour income, we use ONS ASHE data to benchmark hours worked over the life cycle. Chart 4.3 illustrates that for ages 25 to 40, model-generated hours match the data fairly well. Given we do not consider years in education, modelled hours are somewhat higher than the data for households in their early 20s. Labour supply of 50 to 70-year-olds is somewhat underestimated because of trade-offs with respect to matching the asset distribution. Past the age of 70, hours data becomes highly unreliable given small sample sizes. We target a gradual fall off to allow for endogenous labour supply around the pension age but force complete retirement over the age of 80 to prevent adverse incentives, such as hours responding to the bequest motive.[15] With the inflow of foreign capital and a lower average wage, the open economy baseline tends to see higher hours across the age distribution. This ultimately matches the data slightly better than the closed economy solution.

[15] The share of households that could be working that supply zero hours is 4.1 per cent in the baseline. This is the same as the OBR estimate of the long-term equilibrium unemployment rate of 4.1 per cent, but should be interpreted with caution, as the model does not distinguish between inactivity and unemployment and does not have an explicit mechanism for job-search unemployment.

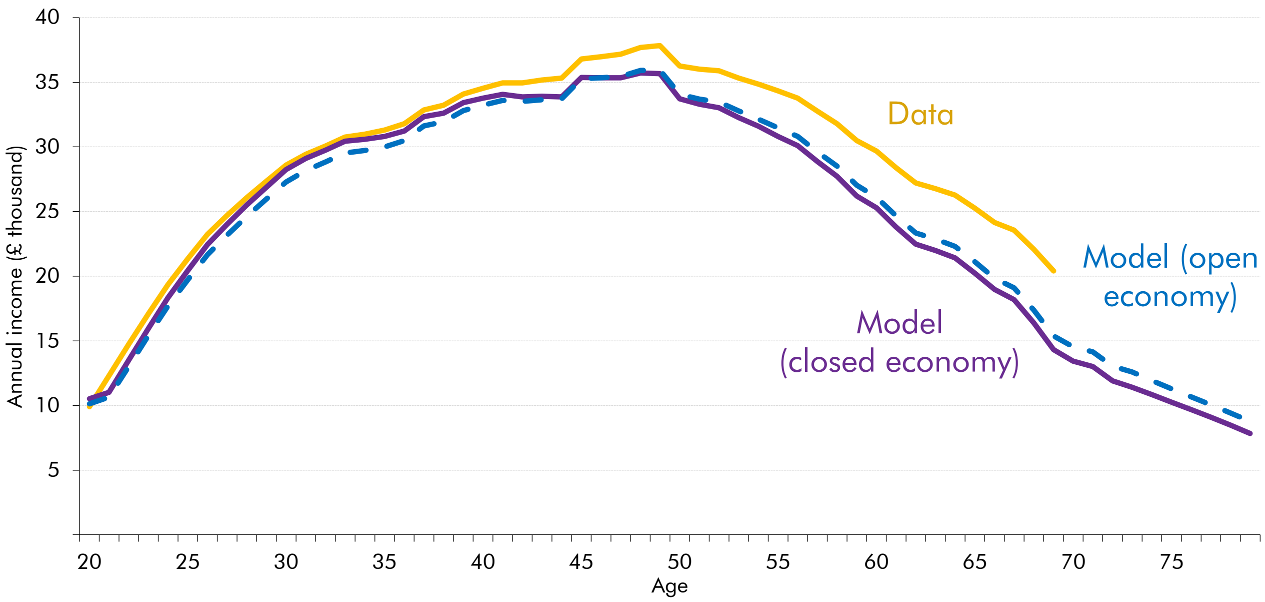
Chart 4.3: Average hours worked by each generation over the life cycle



Source: ONS, OBR

4.9 The distribution of labour income over the life cycle fits the data almost perfectly up to the age of around 50 (Chart 4.4). That is because we feed age-specific productivity profiles directly into the model. For older workers the model somewhat underestimates labour income. Past the age of 70, the data is unreliable and model-generated labour incomes gradually fall before being set to zero from age 80. The open economy baseline is very similar, as productivity and hours changes largely offset each other. The most pronounced differences come at older ages (65 plus), when hours worked are furthest above the closed economy baseline.

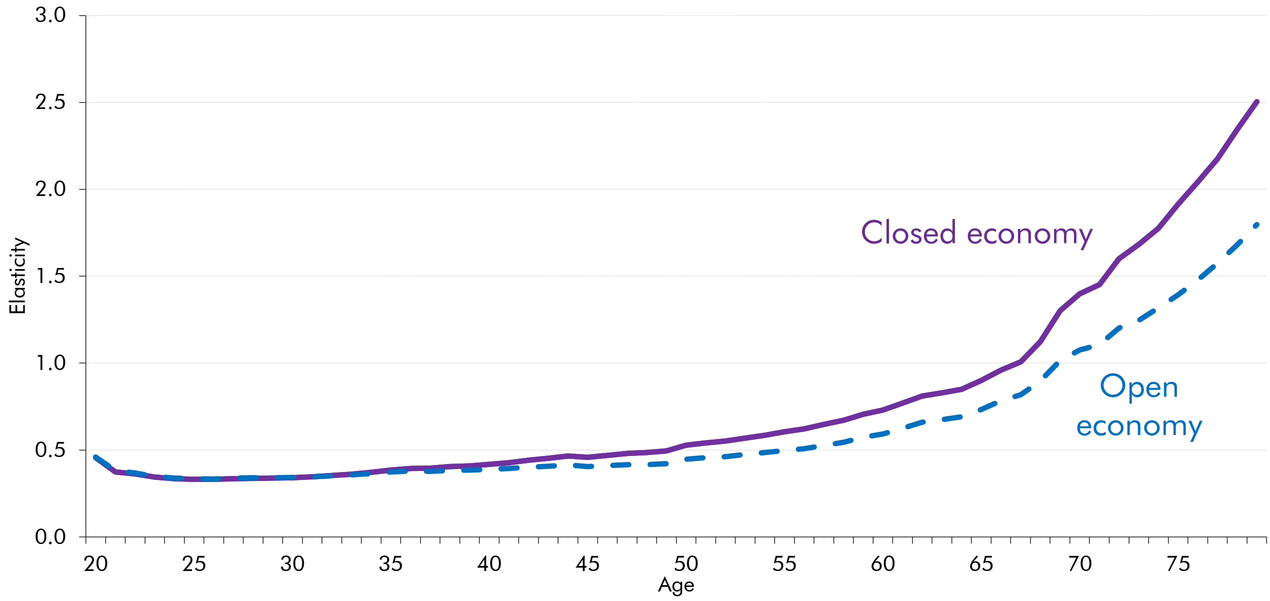
Chart 4.4: Average labour income by each generation over the life cycle



Source: ONS, OBR

4.10 Chart 4.5 plots the Frisch elasticity of hours in response to one-time temporary changes in after-tax labour income. The implied Frisch elasticity is stable between 0.3 and 0.5 until the age of 50. This is similar to estimates for the United States in Congressional Budget Office (2012). The elasticity then rises steeply for older generations, whose labour supply becomes more sensitive to falling individual productivity, state pension receipts and pre-tax labour income. The lifecycle profile is very similar to that calibrated by Cai et al. (2023) for the Australian economy. The upward slope in the Frisch elasticity is less pronounced in the open economy baseline, rising above 0.5 a few years later around age 55.

Chart 4.5: Average Frisch elasticity for each generation over the life cycle



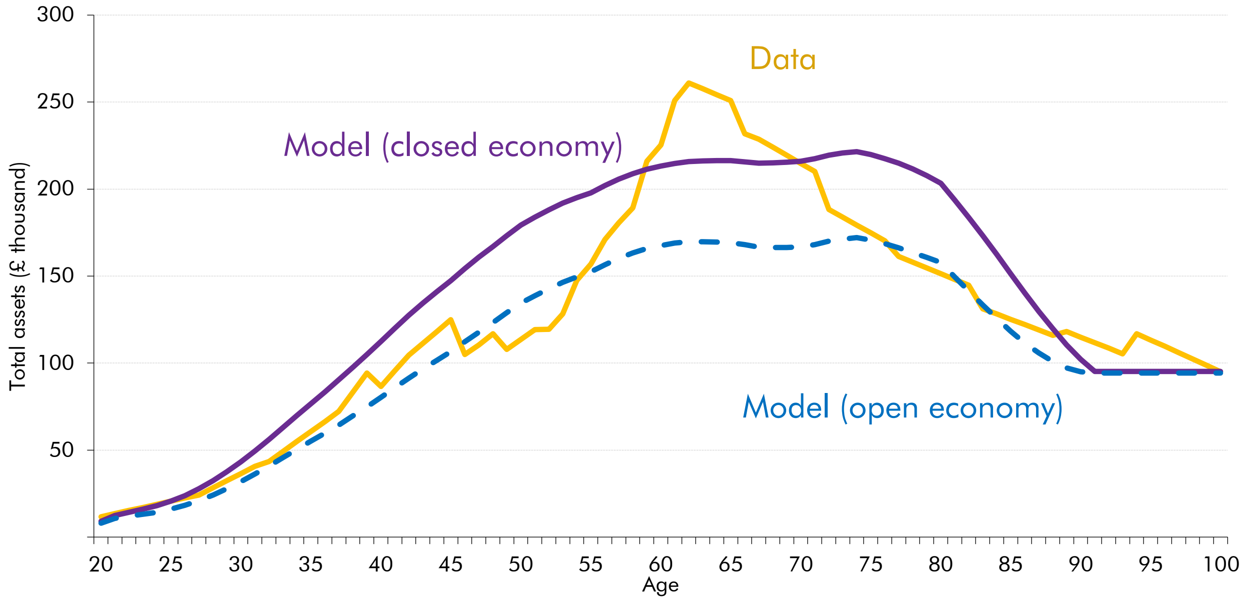
Source: OBR

4.11 Calibrating the model to yield a realistic asset profile over age has proven somewhat difficult. This is because the model is necessarily parsimonious and abstracts from types of assets that are relevant, especially to those at the top of the wealth distribution. There are also data limitations. We use data on the distribution of assets over age from the ONS Wealth and Asset Survey (WAS) as benchmark. Using published information on the profile of wealth over age buckets and the share of housing in total wealth over wealth distribution, we have estimated a life-cycle profile of wealth excluding housing. Net property wealth accounts for 45-50 per cent of total wealth for those in the sixth decile of the distribution and for around 30 per cent of total wealth for those in the top decile, while those in the lowest wealth decile do not hold any positive net property wealth.

4.12 To calibrate the model to the distribution of assets excluding housing, we have adjusted the initial asset distribution of households entering the economy at age 20. We also calibrated parameters for income shocks and their initial values (explained above) with the asset distribution in mind. We set the discount rate to 0.99 to achieve a realistic level of assets across the distribution. To get realistic bequest levels, we calibrate the warm glow parameter ϕ_1, which governs the weight of bequests in the utility function, to 10 such that bequests are nearly all that matters in old age. The target bequest ratio ϕ_2 is calibrated to £107,000 which is consistent with average asset holdings of 90+ year-olds in the data. To ensure asset levels at the end of the age distribution match the data, we also tweak parameter j^*, setting $j^* = 91$, such that the warm glow motive features directly and contemporaneously in households' utility function from that age onwards. This does not mean the model assumes no-one thinks about bequests until age 91, since at much earlier ages there is a probability you will exceed age 91. We set j^* solely as a calibrating assumption to match model outputs with observed life-cycle asset holdings. And since the household value function is based on expected utility across the entire lifetime, the warm glow parameter will begin to form a meaningful part of decision-making well ahead of 91. The asset distribution is further driven by parameters governing earnings shocks.

4.13 Chart 4.6 illustrates that the model generates a realistic life cycle profile for assets in the closed economy baseline. Assets of 20 to 45-year-olds are reasonably well-replicated, but there is some discrepancy between model-generated assets and actual wealth for older generations. We struggle to replicate the full breadth of wealth inequality in the data, and in particular we do not replicate the very high asset levels in the data around the retirement age (mid-60-year-olds). Since part of the capital stock is held overseas in the open economy calibration, asset holdings are lower across the lifecycle, pushing wealth further below the data at the life-cycle peak.

Chart 4.6: Average asset holdings by each generation over the life cycle



Source: ONS, OBR

Macroeconomic aggregates and behavioural parameters

4.14 Parameters of the utility function were chosen as follows. The curvature parameter σ_2 is set to 2 in line with the literature (e.g., the literature review in Cai et al., 2023). σ_1, the weight of consumption relative to leisure, has been calibrated to 0.8 such that the Frisch elasticity is close to one half, given σ_2 and average hours worked, broadly consistent with the literature (CBO, 2012).

4.15 Supply-side assumptions are informed by OBR macroeconomic projections for the UK as published in the March 2024 *EFO*. The rate of labour productivity growth is set to 1.2 per cent, consistent with the OBR assessment of medium-term labour productivity growth. The share of capital in production is set to one third and the long-run population growth is assumed to be around ¾ per cent, in line with the end point of the OBR's March 2024 medium-term forecast. The depreciation rate of capital is set to 5 per cent following the literature (e.g., Cai et al., 2023, Schuster, 2021).

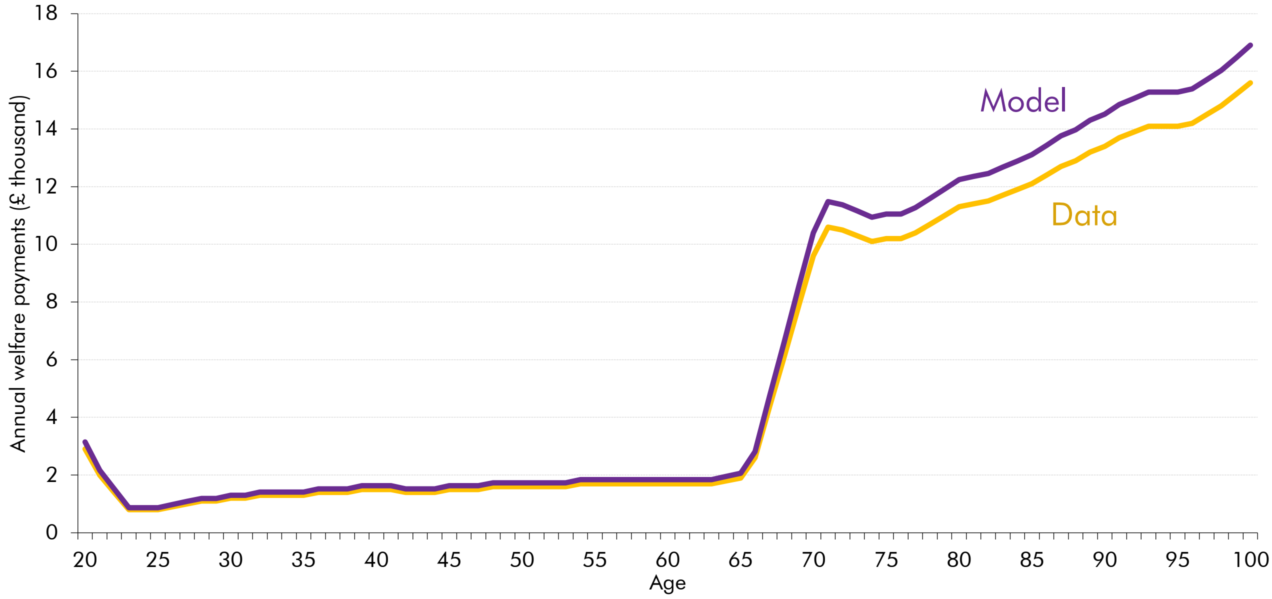
4.16 To assess how closely the model matches reality, we compare key macroeconomic aggregates, that are determined by the model, with the data (Table 4.2). The ratio of private assets to GDP is somewhat lower in the model than suggested by the data, for the reasons explained above. In contrast, the share of consumption in GDP is more than 10 percentage points higher in the closed economy baseline. This is because the model abstracts from other real-world components of expenditure GDP like net trade, housing or government investment. In the open economy baseline, domestic private assets make up just over 55 per cent of the total capital stock.

Fiscal parameters and aggregates

4.17 Table 4.1 (at the end of this section) reports the fiscal parameters we set directly. These include income tax and National Insurance Contributions (NICs) thresholds and marginal rates which we set to their values as of March 2024. Table 4.2 shows that overall income tax receipts are matched well by the model as a ratio of GDP.

4.18 Transfer payments by age are calibrated as follows. We set age-specific scaling parameters γ^j based on information about the distribution of welfare spending over age compiled by the OBR from government statistics. We let the aggregate scaling parameter γ^0 be determined by the model to minimise the distance to an aggregate welfare spending target value which we set to around 10½ per cent of GDP, as observed in the data. Chart 4.7 plots age-specific welfare spending per capita. It shows that during the early years of economic life spending falls as individuals move out of education. It then rises rapidly around the state pension age (currently 66 years). Payments continue an upward trajectory thereafter as uptake of a range of benefits rises in old age, with new benefits available (e.g. pension credit) and the prevalence of ill health rising. Any deviation between the data and the model outputs is a fixed proportional difference, purely due to scaling for GDP per person (which is the same in the open and closed economy baselines).

Chart 4.7: Welfare payments to each generation over the life cycle



Source: ONS, OBR

4.19 Assumptions for the effective rate of Value Added Tax (VAT) τ^c are based on OBR analysis of VAT tax receipts and consumption. Setting $\tau^c = 0.093$ and given a consumption-to-GDP ratio of 75 per cent, model-generated VAT receipts are 7 per cent of GDP, close to the 2022-23 outturn of around 6½ per cent. Pension contribution rates reflect legislated minima and the threshold for income tax relief on pension contributions is aligned with policy as of March 2024. The share of pensions in total household assets excluding housing is estimated from WAS data.

4.20 We set the target ratio for government debt-to-GDP at 100 per cent, to keep the debt stock broadly consistent with current levels. As a result, the residual budgetary item makes up around 6¼ per cent of GDP in the calibrated baseline. It is difficult to compare this with OBR March 2024 *EFO* forecasts directly, particularly since net debt as a share of GDP is lower in those projections than in the UK OLG calibration. However, as a rough approximation, we can separate out the spending and tax revenue categories unmodelled by UK OLG and take the difference between them. Using the fifth year of the March 2024 *EFO* forecast, this estimate of the residual budgetary item is around 14½ per cent of GDP. This is appreciably larger than the model result even though the *EFO* forecast of government debt around 6 per cent of GDP below the UK OLG target.

4.21 Government debt interest payments are generated by the model using model outturns for government debt B and real interest rates r. The closed economy model-generated value of just over 7 per cent of GDP is somewhat larger than recent outturns of around 4½ per cent. The (largely exogenous) open economy value of 5 per cent of GDP is closer to the data.

Table 4.1: Calibrated model parameters

Parameter	Notation	Value	Evidence
Households			
Maximum age	J	81	Economically active age range defined as 20 to 100
Survival probability	s^j		Derived from ONS mortality statistics
Consumption share	σ_1	0.8	Calibrated to Frisch elasticity of 0.5, as in CBO (2012)
Relative risk aversion	σ_2	2	Nishiyama (2013), footnote 31
Individual productivity shifter	κ^j		ONS ASHE
AR(1), persistent earnings shock	ρ^z	0.9	Karahan & Ozkan (2013), Fella, Gallipoli & Pan (2019)
SD, persistent earnings shock	$\sigma_{\epsilon,z}$	0.05	Karahan & Ozkan (2013), Fella, Gallipoli & Pan (2019)
SD, transitory earnings shock	σ_e	0.7	Karahan & Ozkan (2013), Fella, Gallipoli & Pan (2019)
Population growth rate (%)	n	0.75	16+ population growth 2028-29 (OBR March 2024 *EFO*)
Weight on warm glow motive	ϕ_1	10	Calibrated to asset distribution over age from ONS WAS
Bequest target	ϕ_2	4.55	Calibrated to asset distribution over age from ONS WAS
Discount rate	β	0.99	Calibrated to asset distribution over age from ONS WAS
Firms			
Model units to real-world switch	S	19.81	Match GDP per person to OBR March 2024 *EFO*
Productivity level	A	1	Normalised
Share of capital	α	0.33	OBR modelling framework
Labour productivity growth (%)	g	1.2	Trend productivity growth 2028-29 (OBR March 2024 *EFO*)
World real interest rate (%)	r	5.0	Calibrated to OBR forecast of net international liabilities
Depreciation rate	δ	5.0	Cai et al. (2023), Schuster (2021), OBR medium-term retirement rate of business assets
Pensions			
Employee contribution rate (%)	φ^{ee}	5.0	Legislated minimum
Employer contribution rate (%)	φ^{er}	3.0	Legislated minimum
Pension share in HH assets	$pshare$	0.5	Estimated from ONS WAS
Government			
Income tax threshold	TH^1	12,570	Government guidance
Income tax threshold	TH^2	50,270	Government guidance
Income tax threshold	TH^3	125,140	Government guidance
Marginal income tax rate (%)	τ^1	20	Government guidance
Marginal income tax rate (%)	τ^2	40	Government guidance
Marginal income tax rate (%)	τ^3	45	Government guidance
NICs threshold	$NICTH^1$	12,570	Government guidance
NICs threshold	$NICTH^2$	50,270	Government guidance
NICs rate (%)	$eenr^1$	8	Government guidance
NICs rate (%)	$eenr^2$	2	Government guidance
Effective VAT rate (%)	τ^c	9.3	Based on VAT revenues divided by nominal consumption
Transfer distribution over age	γ^j		OBR long-run projections (welfare transfers)
Target debt to GDP ratio (%)	B/Y	100	Approximate current level of public sector net debt to GDP

Source: OBR

Table 4.2: Model outputs

	Per cent, unless otherwise stated		
	Model (closed economy)	Model (open economy)	Actual 2022-23[1]
Private consumption to GDP ratio	74.6	65.5	61.5
Private assets to GDP ratio	376	287	443
Government debt to GDP ratio	100	100	96
Income tax to GDP ratio[2]	9.9	9.2	9.8
Employee NICs to GDP ratio[2]	2.0	2.0	2.7
VAT receipts to GDP ratio	6.9	6.1	7.3
Government debt interest to GDP	7.1	5.0	4.4
Real interest rate	7.1	5.0	3.9
GDP per capita (£ 2019 prices)	35,100	35,100	33,450

[1] Outturn for Private assets to GDP ratio is average of 2018-19 to 2019-20, as this is the latest update of the Wealth and Assets Survey.
[2] Working-age policy ratios are scaled to match the appropriate aggregation of adult versus total population.
Source: ONS, OBR

5 Simulation properties

Overview

5.1 To illustrate the simulation properties of the UK OLG model, this section presents a set of scenarios. These scenarios are designed to show how different aspects of the model react to changes in assumptions relative to a baseline, and therefore to highlight some of the model's key properties and capabilities. The scenarios are generic unit sensitivities, not detailed evaluations of government policy. The results do not constitute the OBR's overall judgement of the impact of each of the scenarios considered – in practice, we would consider a range of models, empirical evidence and the judgement of the Budget Responsibility Committee. We run two tax scenarios, a spending scenario and a demographic scenario to demonstrate a variety of the model's levers.

1. **Scenario A1 (income tax)**: a 1 percentage point increase in the basic rate of income tax.
2. **Scenario A2 (income tax)**: a 1 percentage point decrease in the basic rate of income tax.
3. **Scenario B (state pension)**: a shift in the profile of welfare payments over age by 1 year, broadly consistent with a rise in the state pension age by 1 year.
4. **Scenario C (demographics)**: a 1 percentage point increase in survival probability, broadly consistent with an ageing population.

5.2 The following sections discuss each scenario in turn, focussing on key assumptions and providing an explanation of UK OLG simulation results. Where possible, we also set the model outputs in the context of previous OBR analysis. To assess all levers of the model in full, we employ the closed economy setting and let equilibrium real interest rates be determined endogenously. The steady-state setup of the model means we assume all scenarios take place with enough notice that households can plan their lifetime consumption and working hours with full knowledge.

Scenarios A1 and A2: Changes in the income tax rate

5.3 To show how households respond to tax changes in the UK OLG model, we consider an illustrative 1 percentage point increase or decrease in the basic rate of income tax. Scenario A1 is a rise in the basic rate of income tax from 20 per cent to 21 per cent, while scenario A2 is a reduction in this rate from 20 per cent to 19 per cent. Higher-band marginal income tax rates and tax thresholds remain unchanged from the baseline, as do all other assumptions and parameter settings.

5.4 Changes in marginal tax rates affect the decisions of households in the model because they alter the after-tax income households have available for consumption and saving. Households respond by adjusting their labour supply, consumption and asset accumulation over time. Income tax is levied on labour income, welfare transfers (state pension) and private pension withdrawals. Income tax changes therefore directly affect the decision-making of all generations in the model. Changes in household decisions about labour supply, consumption and asset accumulation in turn have implications for macroeconomic activity, including the level of potential output.

5.5 The model also generates fiscal impacts which arise from direct effects of changing tax rates on tax revenue and the indirect effects from labour supply and household asset profiles. Implementing both a tax rise and tax cut allows us to check the extent to which model outputs are symmetric around current baseline levels of taxation. Model-generated impacts should be interpreted as long-run effects, which materialise once the economy settles into a new steady state. The timing of how long this takes depends on the shock. We would not expect this to take place within the five-year period of the OBR's medium-term forecast, and shocks affecting the structure of the population or age-specific incomes in the model might take decades to reach a new steady state. Long-run impacts of income tax rate changes generated by UK OLG can be cross-referenced against short-run, static impact estimates such as HM Revenue and Customs (HMRC) ready reckoners, and the OBR's assessment of past policies that changed marginal tax rates.

5.6 Simulation results are reported in Table 5.1 as differences relative to the baseline. The increase in the basic rate of income tax (scenario A1) leads to a reduction in GDP of 0.1 per cent relative to the current-policy baseline. Households reduce their total hours worked by 0.2 per cent, but productivity rises by 0.1 per cent as the lowest-productivity workers are the least attached to the labour market. Accounting for offsetting effects from a higher marginal tax rate, but lower hours and labour incomes, we estimate the government raises just over £8 billion in real terms in income tax (around 0.3 per cent of the model's GDP).

5.7 We can see in Chart 5.1 that the reduction in hours is concentrated among those aged over 50, and especially those in the ten years above the pension age. However, since these households work relatively fewer hours in the baseline, these per cent changes translate into small shifts in total hours. At the peak, households aged 72 work on average around 10 minutes less a week. At this point in the lifecycle, households face two competing pressures. On the one hand, they care increasingly about adjusting their assets towards the bequest target. On the other, their age-dependent productivity is lower, so to get the same change in income as a younger worker they must adjust their hours by more. The profile of age-dependent welfare benefits is also significantly skewed towards old age, so these are the only households that have an income without work. That makes participation less attractive, so small changes in their budget constraint can swing the decision on whether to work at all. This is consistent with a higher Frisch elasticity for these generations. Saving as a share of disposable income barely moves and the asset share in GDP dips just 0.2 percentage points. Instead, the drop in post-tax income seems largely to pass through into lower household spending, with the consumption share in GDP falling 0.4 percentage points.

5.8 A cut in the basic rate of income tax by 1 percentage point (scenario A2), leads to largely symmetric results. GDP per person increases by 0.1 per cent as labour supply rises by 0.2 per cent. Productivity falls 0.1 per cent, as the lowest-productivity individuals tend to be drawn back into the labour market by a tax cut. The government raises 0.3 per cent of GDP less from income tax. We can also look at the residual budgetary item, which shows how much room there is for extra spending consistent with a stable debt as a share of GDP. In the tax cut scenario, this falls by 0.3 per cent of GDP, which is very slightly smaller than the 0.4 per cent gain from the tax increase, but the difference is exaggerated by rounding.

Chart 5.1: Simulation results – hours worked relative to baseline (scenarios A)

Source: OBR

5.9 These results from the UK OLG model compare to a direct, short-term revenue impact of £6-8 billion (around 0.2 per cent of GDP) from a basic rate increase of 1 percentage point in the HMRC (2024) tax ready reckoner. The ready reckoner combines a non-equilibrium micro-simulation model result with partial-equilibrium taxable income elasticities. As a result, we might expect it to estimate a larger revenue impact from tax increases compared to a general equilibrium model like UK OLG. However, our model is currently set up with a focus on working adults, with a relatively less developed welfare system. As a result, UK OLG under-estimates the share of individuals with an income below the personal allowance, which pushes up the impact of a change in the basic rate of income tax.

5.10 We can also compare these estimates to recent OBR analysis on the impact of reductions in the main rate of employee National Insurance contributions (NICs).[16] NICs rates are marginal tax rates, but unlike income tax rates they only apply to labour income, and not welfare payments or pension withdrawals. For the 2 percentage point cuts in the main rate of employee NICs announced at each of the Autumn Statement 2023 and Spring Budget

[16] *The labour supply effects of the Autumn 2023 National Insurance contributions cut*, 8 February 2024, and *Economic and fiscal outlook*, March 2024.

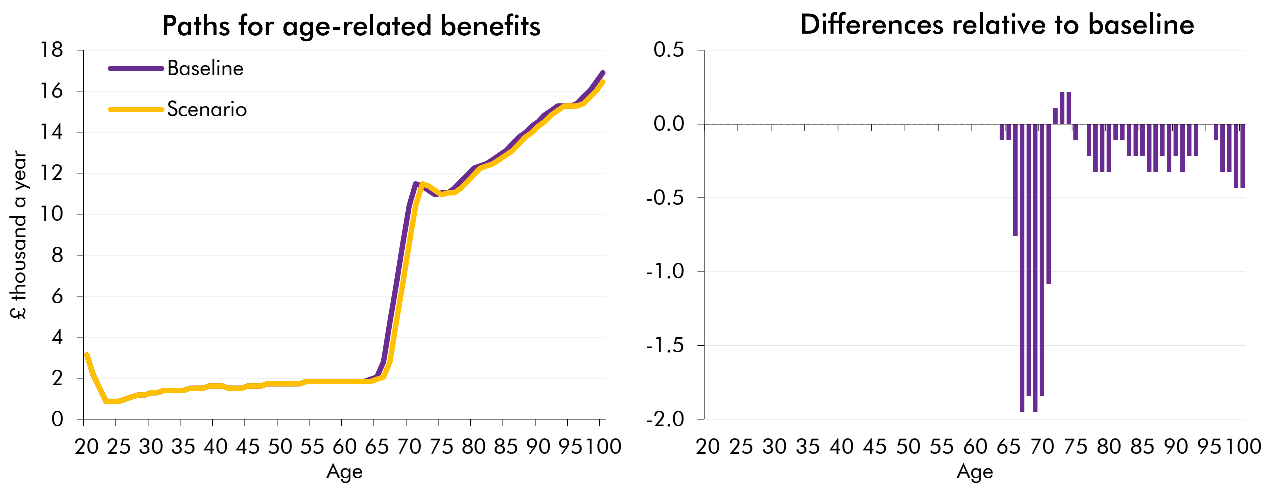
2024, the OBR estimated labour supply would increase by 0.3 per cent.[17] These estimates came from HM Treasury's Labour Supply Model, with specific regard to extensive and intensive margins, along with detailed assumptions about labour supply elasticities that vary by demographic and income groups. UK OLG estimates use less detailed assumptions about the heterogeneity of elasticities across sub-groups of the labour force, but account for macroeconomic feedback effects. Estimates from both approaches are broadly consistent.

Scenario B: Rise in the state pension age

5.11 Scenario B changes the profile of age-dependent welfare spending, which mainly consists of the state pension. We keep the shock generic by shifting the profile of average welfare spending one year over the lifecycle (described in paragraph 4.18). This is broadly consistent with a change in the age from which households are eligible for state pension payments but includes several other effects. In our profile of average welfare spending by age, we are bundling together a range of age-dependent benefits. And the age from which this scenario alters welfare payments is decided by the slope of the payment profile (which starts to rise at age 64) not by a policy parameter.

5.12 In the scenario we lag the age profile of welfare spending by 1 year starting at age 64. The remainder of the profile remains unchanged, including the near-continuous rise in payments after the age of 75 (age 76 in the scenario). This increase is due to the higher uptake of other benefits among older households. We leave all the other assumptions in the model the same as in the baseline, including the productivity profile over age. The change in age-dependent welfare spending alters the trade-off between work and retirement for households near the state pension age. Chart 5.2 shows that households around state pension age see the largest change in welfare payments relative to the baseline.

Chart 5.2: Welfare payments over the life cycle (scenario B)



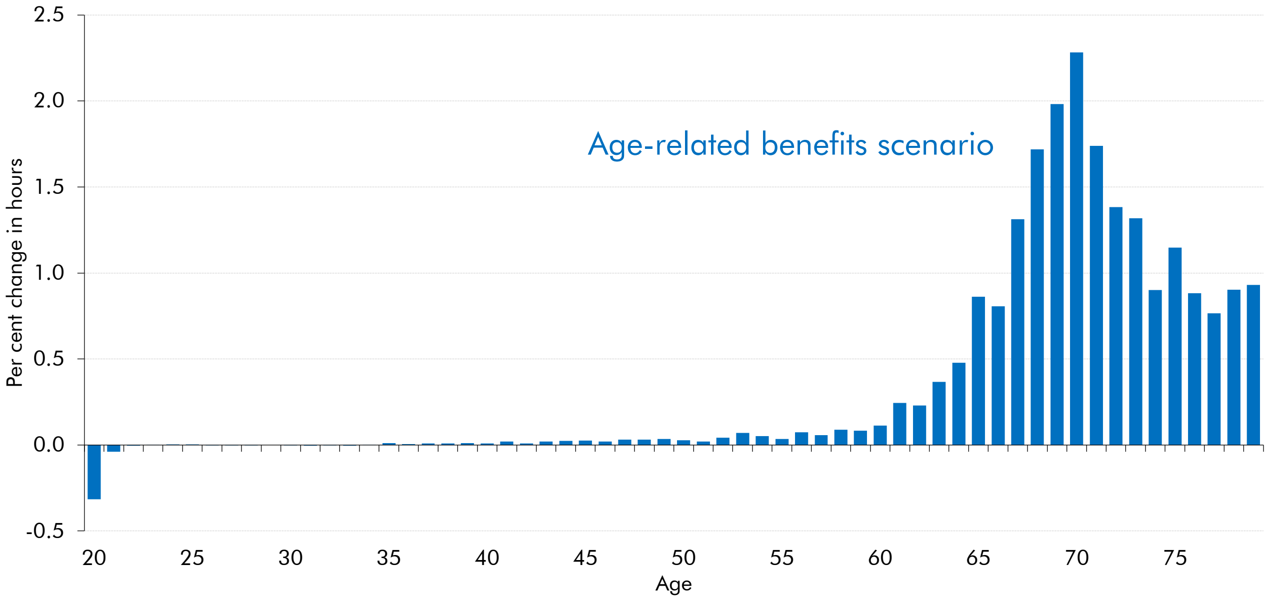
Source: OBR

[17] The NICs policy changes at Autumn Statement 2023 and Spring Budget 2024 differed in their treatment of self-employed workers. The Autumn Statement changes involved a 1 percentage point cut in the primary Class 4 NICs rate paid by the self-employed, along with cancelling the requirement of the self-employed making annual profits above £12,570 to pay Class 2 NICs. Meanwhile, the Spring Budget 2024 measures involved a 2 percentage point cut in the Class 4 NICs rate.

5.13 The delay in welfare payments to older generations in scenario B, broadly consistent with a rise in the state pension age by 1 year, leaves GDP per person slightly higher, raising it 0.1 per cent. This is because households aged over 60 increase their working hours to partly compensate for reduced welfare payments around this age, peaking at a 2.3 per cent rise in hours worked for those aged 70 (Chart 5.3). Yet the change is still modest in absolute terms, amounting to just over 25 minutes a week. Given these households make up a small share of the overall labour force, and their baseline supply of hours is small, this has modest implications for overall labour supply (which rises by 0.2 per cent).

5.14 Spending on welfare decreases around ⅓ per cent, but this fiscal boost is partly offset by weaker income tax receipts. Given the households increasing their hours have lower average productivity, aggregate productivity in the economy falls 0.1 per cent. The net effect of slightly higher labour income, lower welfare payments and unchanged savings withdrawals is that taxable income per person falls 0.4 per cent. This lowers income tax receipts as a share of GDP by 0.1 percentage points, so the overall fiscal improvement is less than the welfare savings at around 0.3 per cent of GDP. Despite a lower consumption-to-GDP ratio, lower post-tax income means that the saving rate is hardly higher than in the baseline. A symmetric scenario that brings age-related welfare benefits forward by one year has largely the same, but opposite, effects.

Chart 5.3: Simulation results – hours worked relative to baseline (scenario B)



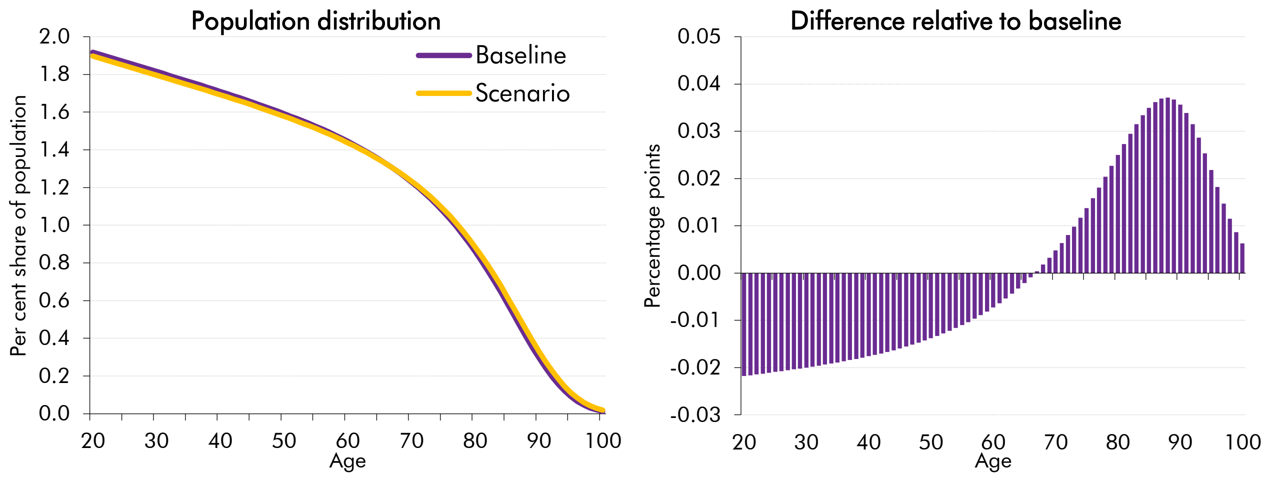
Note: The fall in the youngest workers' hours is related to the downward-sloping profile of welfare payments in the first working years.
Source: OBR

Scenario C: Ageing population

5.15 Scenario C changes the demographic profile of the population. As explained in Section 4, the age profile in the model is determined by mortality rates estimated by the Office for National Statistics (ONS). Chart 5.4 plots the probability of reaching a certain age implied by current mortality rates. The scenario first increases this probability on average by 1 percentage point, with the difference increasing over ages before tapering down after a peak between ages 85 to 90. More tangibly, this could reflect better healthcare raising life expectancy.

5.16 The model then re-normalises the population distribution by age to sum up to 1, lowering the share of young households to compensate for the longer survival of older households. As a result, younger households take up a smaller share of the population, but this effect then reverses for households above around 65. At the peak, households aged 88 now take up almost 0.04 percentage points more of the population. While highly stylised, the scenario is consistent with an older population, higher life expectancy and lower birth rates compared to the baseline. Given age-specific productivity and the timing of state pension payments are unchanged from the baseline, a higher survival probability alters labour supply and capital accumulation over time.

Chart 5.4: Population shares by age (scenario C)



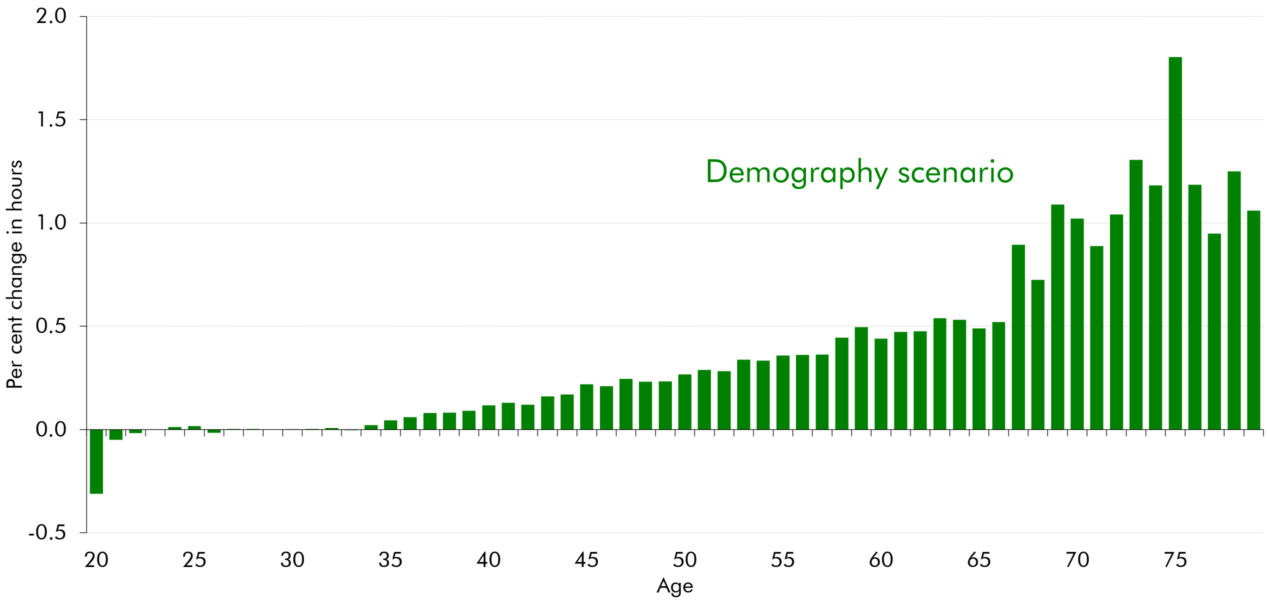
Source: ONS, OBR

5.17 In this scenario, households expect to live longer given the higher probability of reaching old age relative to the baseline. To finance consumption in a longer expected old age, households increase hours worked at most ages. Yet Chart 5.5 illustrates that, from the perspective of each age group, the increase in weekly hours worked is small. Translating the peak proportional change of around 1.8 per cent into actual hours, even those above pension age add less than 20 minutes a week to their labour supply. Meanwhile, from the perspective of the overall population, a larger share of people are in older age groups that either receive the state pension (reducing their incentive to work) or are assumed to work zero hours (once households hit 81). As a result, average labour supply across the whole labour force is 0.5 per cent smaller than in the baseline.

5.18 A higher concentration of households in the more asset-rich stage in their lives means a larger stock of private assets. This raises the capital available to firms and improves productivity, but this is not enough to offset the hours effect. Therefore, output per person is 0.2 per cent lower. With more households alive at the life-cycle peak of wealth, the asset-to-GDP ratio is up by nearly 2 percentage points from the baseline. A larger pool of wealth prompts slightly higher aggregate consumption as a share of GDP, and the increased supply of savings pushes down slightly on real interest rates. But the bigger stock of assets is purely a feature of more households surviving to peak wealth, and in expectation of a longer life the average flow of saving actually decreases.

5.19 With more households living to state pension age, welfare spending is up 0.2 per cent of GDP. Income tax and NICs are largely unchanged, but a more consumption-heavy economy raises more VAT per pound of output. As a result, the overall fiscal position (as measured by the residual budgetary item) worsens by less than the increase in welfare spending, falling 0.1 per cent of GDP.

Chart 5.5: Simulation results – hours worked relative to baseline (scenario C)



Note: The fall in the youngest workers' hours is related to the downward-sloping profile of welfare payments in the first working years.
Source: OBR

Table 5.1: Simulation results (difference relative to baseline)

Variable	Percentage point changes, unless otherwise stated			
	Scenario A1 (IT rise)	Scenario A2 (IT cut)	Scenario B (Welfare)	Scenario C (Demographics)
GDP per capita (% change)	-0.1	0.1	0.1	-0.2
Labour supply (% change)	-0.2	0.2	0.2	-0.5
Labour productivity (% change)	0.1	-0.1	-0.1	0.2
Real wage (% change)	0.0	0.0	0.0	0.3
Real interest rate	0.0	0.0	0.0	-0.1
Saving rate	0.0	0.0	0.0	-0.1
Private consumption to GDP ratio	-0.4	0.4	-0.3	0.1
Private assets to GDP ratio	-0.2	-0.2	-0.2	1.9
Residual budget item to GDP ratio	0.4	-0.3	0.3	-0.1
Income tax to GDP ratio[1]	0.3	-0.3	-0.1	0.0
Welfare spending to GDP[1]	0.0	0.0	-0.3	0.2

[1] Working-age policy ratios are scaled to match the appropriate aggregation of adult versus total population.
Source: OBR

6 Concluding remarks

6.1 The UK OLG model provides an important addition to the OBR's modelling toolkit. It is particularly suited to analyse the impact of economic shocks, economic trends and policies in the long term. Its micro-founded, general equilibrium set-up and ability to analyse household responses along the age dimension allows UK OLG to inform OBR long-run projections and its assessment of the long-run, supply-side impact of policies.

6.2 With the help of illustrative scenarios, this working paper showed that the strength of UK OLG is to study the effects of shocks, trends and policies on households over the life cycle. The main levers provided by the model include taxation of labour income (through income tax and National Insurance Contributions), taxation of income during retirement (through income tax), taxation of consumption (through VAT), pension incentives (tax relief), welfare spending (mainly the state pension), and structural shifts that include changes to the demographic profile of the population, changes in behavioural parameters, or changes in underlying macroeconomic and fiscal parameters. A change in these parameters triggers an age-dependent behavioural response of households who adjust their labour supply, consumption and saving decisions across their full expected lifetime. Any household response has macroeconomic and fiscal implications in general equilibrium, including an adjustment in the capital-labour ratio, equilibrium real interest rates or the long-run financing requirement for government. The results from our model look reasonably similar to existing external estimates, including ready reckoners already in use.

6.3 The UK OLG model can serve as the basis for future extensions that may be tailored to specific analytical and policy questions. It can act as a transparent and tractable core model that will continue to be developed and extended by OBR and other government staff. As such, it necessarily omits some real-world complexities that will matter for specific questions in the future. Limitations of the model in its current core form include the following:

- UK OLG improves on policy OLG models at other fiscal institutions by including stochastic shocks to income. This generates heterogeneity in income and assets within generations which has macroeconomic implications by giving rise to precautionary saving. The degree of income and wealth inequality, however, remains fairly abstract, with limited scope to capture interactions between individual productivity, working-age benefits and labour supply. A future extension could follow the literature (Cai et al., 2023; Schuster, 2021; d'Andria et al., 2020) and explicitly model different skill and income groups within each generation of households. This could improve how well the model reflects real world inequalities and impacts of tax and welfare policies on working-age households.

- Unlike other OLG policy models (e.g., Cai et al., 2023), UK OLG captures the production sector in very simple terms, abstracting from multiple sectors of the economy and supply chain interlinkages. It also refrains from explicitly modelling international trade and capital flows. For questions aimed at understanding shocks to individual sectors, for example around climate change mitigation and adaptation, or analysis of the UK current account, more detail could be added to the production and international side of UK OLG.

- We chose to model bequests in a simplistic way to ensure tractability and reasonable run times while being able to match the key moments of household assets data in the UK. Adding further detail to how bequests feature in the utility function and are allocated to surviving generations could improve the model's ability to address certain policy questions, including regarding inheritance tax.

- We have solved this version of the model using a budget rule where the government holds debt as a share of GDP stable. In future iterations, we can explore using different conditions on the government to solve the model.

- UK OLG involves relatively thin financial markets, with only one asset type and equal private and government interest rates. While differentiating safe and risky assets would require a wide-ranging rebuild to include aggregate uncertainty, a wedge between private and government interest rates could be an area for future development.

6.4 Staff at the OBR welcome suggestions on how to improve the UK OLG model further. Please send suggestions to feedback@obr.uk.

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