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A small model of the UK economy

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Abstract

This paper presents a small calibrated New-Keynesian model of the UK economy extended to take account of credit risk premia and unconventional monetary policy. It can be used to run simulations or provide alternative economic scenarios – in terms of differences from the Office for Budget Responsibility's central economic forecast. The model can be applied to provide some indication of how the stance of monetary policy might adjust in response to fiscal policy announcements.

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JEL references: E12, E13, E17, E3, E43, E47, E52

Keywords: Macroeconomic model, New Keynesian IS-LM model, economic forecasting, simulation, monetary policy, fiscal policy.

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1 Introduction

1.1 In 2010, the Office for Budget Responsibility began to publish illustrative economic scenarios alongside its central forecast. Since then, the tools used to produce them have been developed. In this paper, I present a simple calibrated model of the UK economy which can be used to produce alternative forecasts, to conduct simulations and as the framework underpinning the OBR's scenario analysis.

1.2 Section 2 describes the theoretical structure of the model, beginning with a standard New-Keynesian model and describing how the model presented here differs from it. Section 3 explains how I have extended this model to allow for the effects of credit risk premia and unconventional monetary policy. It also provides a very simple characterisation of the dynamics of the public finances. Section 4 describes the approach taken to calibration and sets out the simulation properties of the model. Section 5 looks at one simple application of the model to a forecasting issue – how monetary policy might be expected to respond to a series of anticipated fiscal policy shocks.

2 A four-equation model

2.1 There are a number of ways in which to develop a model of the economy, the suitability of which depends upon its intended use. The sorts of models produced by academic macroeconomists typically adhere strictly to the prescriptions of their microeconomic foundations. That is to say that, whether it's a model with three equations or twenty, the laws of motion of the economy are governed by the optimising behaviour of agents operating within it. These models are primarily concerned with the assessment of policy, and often, the appraisal of what constitutes optimal policy. The policy prescriptions that follow from such an approach depend critically on the internal consistency of the models used and the structural stability of those models in the face of changes to policy regimes.

2.2 There exists a different class of models, such as the large-scale macroeconomic model used by the Office for Budget Responsibility, that is primarily concerned with forecasting.[1] These models are not usually derived from microeconomic foundations and rely more heavily upon a good fit with the data. It is widely known that such models are vulnerable to the Lucas critique because the behavioural relationships observed in the data cannot be considered structural and could change when the operation of policy changes.[2] This is a problem for policy analysts but less so for forecasters, who are not typically asked to forecast the effect of significant changes to the macroeconomic policy framework.[3]

2.3 The OBR's main macroeconomic model allows a forecaster to apply judgement in a structured and consistent way. That judgement can improve the accuracy of forecasts is well-documented and, therefore, is an important feature of the approach taken.[4] The size and scope of the main model are also vital for generating forecasts of the numerous variables required to undertake a rigorous fiscal forecast.

[1] See OBR (2011) for more details on the model and the forecast more generally.

[2] See Lucas (1976).

[3] Nevertheless, forecasters rely on models fitted to past data and care must be taken to identify whether that data may have been generated by a different policy regime.

[4] Wallis and Whitely (1991), Clements and Hendry (2002) and Groen et al (2009) all explore the application of judgement to forecasts.

2.4 The model presented here is not designed for the same purpose. Its intended use is to provide quick and easily-understood forecasts of core variables without the application of judgement, which can readily be presented in terms of differences from the central forecast. This facilitates the simple assessment of alternative economic scenarios.

2.5 Given the trade-off between models that replicate the inertial responses of output, inflation and interest rates and those which most closely adhere to the forward-looking behaviour expected of agents in the economy, it stands to reason that the model with the greatest practical flexibility might be one which draws from both of the classes described above. This issue is described at length in Pagan (2003).

2.6 Given its intended use, the model presented here sits broadly between the two classes of model described above - borrowing the more desirable features of micro-founded models but relaxing some of the stricter assumptions to allow a better fit with the data.

2.7 There is always a temptation to expand any model because it is desirable to identify the effects of different types of shock and a larger model permits a wider assessment of these. However, a smaller model need not imply a less accurate forecast. [5] And its size can be a strength; smaller models make it easier to trace the transmission of shocks through the model onto key variables, and it is also easier for the modeller to experiment with alternative assumptions, such as the degree of forward-looking behaviour that agents exhibit.

2.8 The four key variables of interest are the output gap, Bank Rate, the inflation rate and the exchange rate; given by the investment-saving (IS), Taylor, Phillips and uncovered interest parity relations respectively. I also include an equation that captures changes in the cyclical component of the government's primary balance. In what follows, I set out the functional forms adopted for the core equations and identify where the assumptions are consistent with the microeconomic theory upon which they are founded and where they deviate from it. For reference, a complete set of the equations that constitute the model can be found in Annex C.

[5] Del Negro and Schorfheide (2012) assess the forecast performance of the medium-sized Smets and Wouters (2003) DSGE model, augmented to include the effect of financial frictions, and a small scale DSGE model – more like the one presented in this paper. They find that while the short-term forecast performance of the SW is slightly better than the small-scale model, the medium-term forecast performance of the small-scale model is slightly superior.

IS relation

2.9 The IS equation relates output in the economy to deviations of the real interest rate from the level consistent with stable output and inflation in the medium term. Equations of this form are a staple of macroeconomic modelling and appear, in some form, in all New-Keynesian models. The forward-looking IS relation is given by,

$$c_t = \beta_c c_{t+j_{|t}} + \beta_r z_{t+j_{|t}} + \mu_t . \qquad (2.1)$$

2.10 Equation 2.1 represents the baseline consumption Euler equation that arises from the representative household's optimisation problem. It has been log-linearised around its steady state so c_t represents the deviation of consumption from its steady-state growth path, $c_{t+j|t}$ is the expected deviation of consumption from its steady state, conditioned on information available at time t, $z_{t+j|t}$ is the expected real interest rate gap conditioned on information available at time t and μ_t is an independent, identically-distributed consumption shock.

2.11 The consumption Euler equation simply states that, in equilibrium, the representative household is unable to increase its utility by shifting consumption between periods – that is, the marginal utility of consumption today is balanced with the discounted marginal utility of consumption tomorrow.

2.12 To get from the consumption Euler equation to the IS equation I assume that the behaviour of the consumer can explain whole-economy behaviour. This is a common assumption in small models of the economy but is not completely satisfactory given, in particular, the contribution of business and inventory investment to the cyclical volatility of output.

2.13 Without deriving the behaviour of firms explicitly from microeconomic foundations here, it suffices to say that the change in output associated with firms' responses to changes in real interest rates is in the same direction as that implied by the response of households. Intuitively, if the real rate of interest falls, this lowers the cost of borrowing and increases the overall rate of return of an investment project. Therefore, any profit-maximising firm has a greater incentive to invest.[6]

[6] Tobin's Q theory of the investment decision, Tobin (1969), operates in a similar way. Lower expected interest rates decrease the rate at which income streams are discounted, increasing the valuation of companies' net assets. When the market value of assets exceeds the book value, there is a profit opportunity and companies expand their investment until such a time that book prices are equal to market prices.

2.14 There are a number of extensions to these simple theories, which highlight the role of uncertainty and irreversible costs in the investment decision - see Leahy and Whited (1995) and Pyndick and Solimano (1993), for example. However, these are beyond the scope of this paper. While the effects of uncertainty are not included in this model, one of its features is that the cyclicality of business investment and its contribution to output volatility should already be captured in the IS relation, to the extent that the investment decision is dependent on the real interest rate gap.

2.15 Aggregating the consumption Euler equation to the whole economy level gives equation 2.2. I also include a term for changes in the trade-weighted real effective exchange rate, which is intended to capture the effect on output of changes in relative prices which serve to shift the allocation of resources to and from the export-facing sector,

$$y_t = \beta_y y_{t+j_{|t}} + \beta_r z_{t+j_{|t}} + \beta_{er} \Delta er_{t+j_{|t}} + \mu_t . \qquad (2.2)$$ [7]

2.16 y_t is the output gap, $y_{t+j|t}$ is the expected output gap at time t and $z_{t+j|t}$ is the expected real interest rate gap, $er_{t+j_{|t}}$ is the real expected exchange rate and μ_t is an independent and identically-distributed aggregate demand shock.

2.17 Having presented the baseline IS equation, it is worth commenting on its strengths and limitations. The strength of this approach is that the output equation is based on optimising behaviour and so should be robust to changes in policy – it is characterised by 'deep' or structural parameters. Generally, criticisms fall into two categories, concerns over the application of representative agent theory and the assumption of rational expectations.

2.18 The development of models based on the optimising behaviour of a representative agent was prompted by concerns raised by Lucas (1976) about the traditional approach to macroeconomic modelling – which depended on an assumption of stability in the statistical relationships between variables. The representative agent approach is a simplification that saves having to capture the complex behaviour of millions of individual agents. Such models are now commonplace but most ignore the heterogeneity of agents in the economy, which may have a significant bearing on aggregate behaviour.[8] I abstract from this consideration here while accepting that the validity of aggregation contributes to

[7] See Gali & Monacelli (2005) for a detailed derivation of the open economy IS curve under domestic inflation targeting.

[8] For critiques of representative agent theory see Kirman (1992) and Hartley (1996).

the already significant uncertainties associated with any small model, which necessarily makes dramatic simplifications of reality.

2.19 The second criticism of the IS relation concerns the assumption of rational expectations. Estimated forward-looking IS relations tend to underperform simple autoregressive models and, quite often, little empirical support is found for the relationship between output and expected real interest rates.[9] This is partly because the output gap moves with a degree of inertia that is inconsistent with the adjustment paths implied by forward-looking, rational expectations models. In these models, it is the rational but immediate adjustment of households' expectations to innovations which implies a jump response in consumption, which, in practise, is rarely seen in the data.

2.20 Some attempts have been made to explain why consumption reacts so slowly in response to changes in interest rates. One such endeavour is the habit formation model of Fuhrer (2000). Fuhrer postulates that the utility derived from consumption depends both on the absolute level of consumption and the level of current consumption relative to past consumption – that households do not like consuming less than they have been and initially resist changes, before eventually adjusting. This modification was shown to substantially improve the fit of the model.[10]

2.21 Other work, predominantly concerned with why the behaviour of consumption appears to invalidate the permanent income hypothesis, such as Muellbauer (1988), suggests that households may be myopic in their consumption choices. Campbell and Mankiw (1989) offer the hypothesis that households do not have the resources to engage in producing full forecasts and so it is optimal for them to use a rule of thumb when updating their consumption plans in response to income shocks.

2.22 That lagged output improves the fit with the data is important, but whether one accepts the habit formation story, the rule of thumb hypothesis or simply assumes that households are less forward-looking than is often suggested, is less important for the specification of the IS relation. In empirical work, an assumption of habit formation or myopia in household consumption choices is not uncommon and both Batini and Haldane (1999) and Smets and Wooters (2003) allow for it in their respective models of the UK and the euro area economies. Indeed, neither Batini and Haldane nor Carlin and Soskice (2010) include expected output in their baseline IS relations, an approach which I follow

[9] See Fuhrer and Rudebush (2004).

[10] See, for example, Giannoni and Woodford (2003) for a formal derivation of the habit formation-augmented NKIS relation.

here. Therefore, the baseline IS relation employed here includes lags of output, real interest rates and exchange rates but not expectations thereof,

$$y_t = \beta_y y_{t-j} + \beta_r z_{t-j} + \beta_{er} \Delta er_{t-j} + \mu_t . \tag{2.3}$$

Phillips curve

2.23 The New Keynesian Phillips Curve (NKPC) relates current inflation to expectations of future inflation and marginal cost pressures. That the inflation process is forward-looking follows from the price-setting behaviour of firms, which is assumed to follow Calvo (1983). The basic premise is that in each period a firm has a fixed probability that it will keep its price unchanged, so firms set prices now with a view to the future because they know that they may not be able to change their prices in the subsequent period.[11] The probability of changing/not changing price each period is independent of the time elapsed since the firm last changed its price, and this attribute simplifies the aggregation of individual firm behaviour to the whole-economy level. This gives an equation of the form,

$$\pi_t = \lambda_\pi \pi_{t+1|t} + \lambda_y y_t + \varepsilon_t \tag{2.4}$$

where π_t is the rate of inflation and $\pi_{t+1|t}$ is the expectation of inflation conditioned on information available at the current time.

2.24 I assume that real marginal cost pressures drive the inflation process, consistent with Gali and Gertler (1999) and that these cost pressures are well-represented by the output gap, y_t . There are other measures which could be used – Batini et al. (2005) use the labour share of income in their estimate of the Phillips curve, which has the advantage of being directly observable.[12] But using the labour share for forecasting with this model would not be possible because it does not capture the evolution of the labour market, so the output gap is preferred. The error term, ε , is an independent, identically-distributed inflation shock.

2.25 As with the IS relation, the purely forward-looking version of this equation fits the data poorly – failing to capture the observed inertia of inflation. The equation specification implies that a high degree of persistence in either movements in

[11] Note that this probability is independent of the general level of inflation. This seems unlikely, and has implications for the model, such as the potential non-neutrality of money.

[12] It is also the case that, under certain assumptions, the labour share (the average product of labour) is proportional to real marginal cost in an economy characterised by a Cobb-Douglas production function.

the output gap or changes in inflation expectations could produce an inertial path for inflation, but leaves open the possibility of large jumps. It also implies that inflation should lead the output gap, which is the opposite of what we observe in the data; both empirical evidence and conventional wisdom suggests that monetary policy affects inflation only with a lag, rather than instantaneously.

2.26 A model that does not adequately capture the persistence of inflation would be of little use to forecasters. Therefore, in what follows, I relax the restrictive assumption that households and firms are completely forward-looking. This approach is consistent with the approach I have taken to expectations in the IS relation.

2.27 The hybrid version of the New Keynesian Phillips Curve, used in a number of empirical estimates of the equation (Gali and Gertler (1999)) modifies the standard NKPC formulation by allowing a proportion of firms to use a rule of thumb when setting prices. This modification provides a theoretical justification for the presence of an inflation lag in the first order condition of the NKPC. Intuitively, the inclusion of lags of inflation serves to act as a proxy for the rational expectation of future values of the driving variable. The resulting equation therefore includes a backward-looking term and a coefficient, ς, that determines the weight placed on past inflation relative to inflation expectations in the inflation process,

$$\pi_t = \varsigma\pi_{t-j} + (1-\varsigma)\pi_{t+j_{|t}} + \lambda_y y_{t-j} + \varepsilon_t. \tag{2.5}$$

2.28 The restriction placed on the inflation coefficients summing to unity (effectively imposing a discount factor of one) means that money is super-neutral in this model. It also implies that the coefficient ς can be interpreted directly as the proportion of firms in the economy that set prices in a backward/forward looking manner.

2.29 In this paper I take a slightly different approach to the Gali and Gertler set-up and adopt the prior expectation that agents in the economy expect that monetary policy is able to return inflation to target at some time horizon (typically assumed to be around two years). Therefore, I use equation 2.6 and set $\beta\pi_{t+1_{|t}}$ equal to the inflation target, π^*,

$$\pi_t = \varsigma\pi_{t-j} + (1-\varsigma)\pi^* + \lambda_y y_{t-j} + \lambda_e \Delta e_{t-j} + \varepsilon_t. \tag{2.6}$$

2.30 To allow for the effect of exchange rate pass-through to prices, I include the change in the trade-weighted nominal effective exchange rate. That the Phillips curve can be augmented in this way is demonstrated formally in Batini et al. (2005).

Uncovered interest parity condition

2.31 There is a vast literature surrounding the performance of models concerned with forecasting movements in exchange rates. In their seminal paper, Meese and Rogoff (1983) show that the forecast performance of economic models of the exchange rate is typically worse than simply assuming the future exchange rate will be whatever it is at the moment – a random walk model.

2.32 A more recent study, Cheung et al (2005), compared the forecast performance of a wide range of models put forward in the 1990s and showed that little progress has been made. Emphasising that while some structural models appear to outperform a random walk model at very long horizons, short-term movements remain unpredictable. This finding reaffirms the Obstfeld and Rogoff (1995) observation that deviations from absolute purchasing power parity - which simply states that the prices of tradable goods and services across countries should be equal - can persist for decades.[13] Furthermore, Hauner et al (2011) find that the forecasts produced by a number of models of exchange rates are only weakly correlated with 'Consensus' forecasts of the exchange rate.

2.33 With this in mind, I make no serious attempt to forecast the exchange rate here, beyond the simplest possible specification of a no arbitrage condition – uncovered interest parity (UIP). Notwithstanding these forecasting difficulties, the exchange rate has an important bearing on economic developments: the recent history of the sterling exchange rate is one of prolonged periods of relative stability, punctuated by relatively rapid and substantial adjustments (in 1992, 1996-7 and 2008-9), and I therefore include it in the IS and Phillips relations. This should improve the model's short-term unconstrained forecast, but also permit the simulation of exchange rate shocks.

2.34 Equation 2.7 is the UIP condition that gives the forecast interest rate, which is consistent with the equation used in the OBR's main macroeconomic model,

$$e_t = e_{t-1} + i_t^f - i_t + \wp_t \qquad (2.7)$$

[13] Which is why, they suggest, that movements in the exchange rate should not appear in a monetary policy rule.

where e is the log nominal exchange rate, i^f is the foreign interest rate, i is the domestic policy rate and $\wp$ is an independent and identically-distributed exchange rate shock.

2.35 The equation is based on the idea that, if an interest rate differential exists, the nominal rate of return on domestic and foreign assets is equalised by movements in the nominal exchange rate.[14] In this model foreign interest rates are assumed to be exogenous and, in steady-state, are equal to the steady-state domestic nominal interest rate.[15]

2.36 For the purposes of including changes in the real exchange rate in the IS relations it is necessary to have a forecast of foreign inflation. This is input to the model exogenously and assumed to have a steady-state rate consistent with the domestic inflation target. This, combined with the interest parity condition, ensures the stability of the steady-state real exchange rate. This is a similar assumption to that made by Carlin and Soskice (2010).

2.37 For the purposes of augmenting the IS relation, the change in the real exchange rate is given by the identity,

$$\Delta er_t = (e_t - e_{t-1}) + \pi_t - \pi_t^f \tag{2.8}$$

where the change in the log real exchange rate Δer_t is given by the change in the log nominal exchange rate, $e_t - e_{t-1}$, and the relative rate of inflation. Foreign prices are an exogenous input to the model.

Central bank reaction function

2.38 Taylor (1993) observed that the conduct of monetary policy can be well-captured by a simple rule relating interest rates to inflation and the output gap. Following Taylor's paper there began a concerted academic effort to assess this class of policy rules and their implications for optimal monetary policy. However, some form of Taylor's original rule, which is entirely backward-looking, remains the default specification for the behaviour of the central bank in many economic models.

[14] Note that the UIP condition is only an argument about the expected *change* in the exchange rate – it does not pin down its level.

[15] This assumption seems reasonable given that, for example, the US and the euro area are approximately targeting the same medium term inflation rate, and exhibit broadly equivalent productivity growth.

2.39 The IS and Phillips relations described above operate with a lag. That is to say, it takes time for interest rates to affect the output gap and, in turn, for inflation to respond to the output gap. The lag structure embodied in these equations means that monetary policy should be conducted with a view to the future. Therefore, given the involvement of the Bank of England in forecasting the economy and the lags associated with the conduct of policy, I specify a forward-looking form of the Taylor rule which is consistent with the other equations in the model – the Bank's expectations are assumed to be model-consistent.

2.40 As well as being a reasonable empirical description of the conduct of monetary policy, Svensson (1997) and others have shown that the Taylor class of rules can also be derived from the inflation targeting central bank's optimisation problem. Simply allowing for the lag structure associated with the monetary transmission mechanism gives the forward-looking Taylor rule specified in equation 2.9,

$$i_t = \bar{i}_{t-j} + \gamma_y y_{t+j_{|t}} + \gamma_\pi \left(\pi_{t+k_{|t}} - \pi^*\right) \tag{2.9}$$

where i_t is Bank Rate, $\bar{i}_{t-j}$ is the equilibrium nominal rate of interest, $y_{t+j_{|t}}$ is the output gap forecast at the relevant time horizon and $\pi_{t+k_{|t}} - \pi^*$ is the forecast deviation of inflation from target.[16,17]

2.41 Unlike the IS and Phillips relations, I do not include an exchange rate term in the specification of the Taylor rule. In this model, the central bank responds to movements in the exchange rate only indirectly, via its effect on output and domestically-generated inflation. This is consistent with the Taylor (2001) finding that the inclusion of exchange rates does little to improve the stabilisation of output and inflation and is possibly detrimental.

2.42 A substantial literature also exists on the observed inertia of interest rate setting by central banks around the world, see for example Goodfriend (1991). In what follows, I adopt the same approach as Clarida, Gali and Gertler (1999), which is to assume the presence of a policy rate smoothing parameter in the central bank's reaction function. They suggest this smoothing arises from a desire to avoid the credibility costs associated with large policy reversals, a desire to

[16] The specification is slightly different from the original Taylor rule but consistent with Nelson and Nikolov (2002).

[17] I use effective bank rate in place of actual bank rate to account for the effects of credit spreads and unconventional monetary policy on lending rates to the wider economy. I discuss this in more detail from section 3.1.

minimise disruption to capital markets and the time it takes build a consensus to support a policy change.[18]

2.43 Later discussions have identified ways in which interest rate smoothing might be optimal for a central bank in the presence of parameter uncertainty. Svensson (1999), for example, shows that parameter uncertainty for an inflation-targeting central bank dampens the policy response, confirming what Brainard (1967) first described. Soderstrom (2002) extends this analysis to a dual-mandate central bank with output in its loss function. He finds that uncertainty over inflation dynamics tends to heighten the response to inflation deviations (in case expectations become unanchored) but uncertainty over output dynamics encourages caution.

2.44 Regardless of the precise motive, the inclusion of central banks' smoothing of policy rates in their reaction functions significantly improves the fit with the data. Equation 2.10 captures interest rate inertia as in Clarida et al (1999),

$$i_t = (1-\psi)i_t^* + \psi i_{t-1} \qquad (2.10)$$

where i_t is the interest rate set, ψ is the smoothing parameter, i_t^* is the interest rate implied by the reaction function (absent smoothing) and i_{t-1} is the interest rate set in the preceding period.

2.45 Substituting the generalised Taylor rule in to equation 2.10 as the i_t^* term gives the central bank reaction function with policy rate smoothing equation 2.11,

$$i_t = (1-\psi)\bar{i}_t + (1-\psi)\gamma_y y_{t+j_{|t}} + (1-\psi)\gamma_\pi\left(\pi_{t+j_{|t}} - \pi^*\right) + \psi i_{t-1}\,. \qquad (2.11)$$

2.46 The functional form presented in equation 2.11 implies that the central bank is more forward-looking than other agents in the economy. This assumption is consistent with the observation that central banks spend a substantially larger share of their resources attempting to forecast the evolution of output and inflation than do households or firms. This choice of modelling approach implies that the central bank could exploit the adaptive expectations of private agents - the classic time-inconsistency problem.

[18] In a rational expectations context, Woodford (2003) shows that it can be optimal for a central bank to move the current policy rate less in response to demand and inflation shocks if, at the same time, the changes are characterised by a high level of persistence. Such an argument critically depends upon the forward-looking expectations of agents in the economy, which is not a feature of this model.

2.47 This is one of the inevitable drawbacks of building a model that retains some use for forecasting. However, the primary purpose of this model is not to assess the effects of monetary policy regime change. The macroeconomic framework is well-established in the United Kingdom and this supports the validity of using a model of this type.

3 Extensions

Credit spreads

3.1 Since the onset of the recent financial crisis, a rise in the perceived degree of risk associated with lending and borrowing has significantly widened the gap between the interest rate set by the Bank of England and the price of credit available to the wider economy. As a result, monetary policy has subsequently taken serious account of the effect of credit spreads on the behaviour of agents in the economy.

3.2 The inclusion of credit spreads in the model presented here is based on a simple principle: the Bank of England is ultimately concerned with the interest rates paid by household and firms in the economy. So if the spread of interest rates experienced by agents in the wider economy over policy rates is 200 basis points higher than usual, this implies that the Bank would set policy rates around 200 basis points lower than usual. Therefore, rather than targeting policy rates, in this model, the Bank takes credit spreads into account directly and targets an adjusted policy rate, described here as Effective Bank Rate, i_t^e.

3.3 By extending the New-Keynesian model of the economy to include a measure of credit spreads, Curdia and Woodford (2009) show that agents in the economy respond in a similar fashion to increases in borrowing rates arising from changes in the default risk premium as they would to an increase in Bank Rate.[1] Importantly, the Curdia-Woodford model shows that, so long as central bankers take credit spreads into account, the Taylor class of policy rules remains optimal in choosing the stance of monetary policy.

3.4 To construct a measure of the credit spread, I use a selection of quoted household borrowing and deposit rates and subtract from those the relevant reference rate of interest.[2] For example, I take the average interest rate quoted for

[1] Curdia and Woodford create a model which assumes that banks are able to finance themselves by issuing deposits which must attract the same rate of interest as government bonds of the same maturity to avoid arbitrage opportunities. In this paper I assume that the relevant spread is over the cost of borrowing, as set by the central bank. This approach is motivated by the observation that the Bank targets a policy rate defined in terms of very short-term government borrowing rates – the gilt repo rate. And QE is conducted by buying and selling government bonds.

[2] Ideally, a measure of credit spreads would also include corporate sector borrowing and deposit rates, but there is little data available with which to construct such a measure.

a 2-year fixed-rate mortgage and subtract from this the two-year gilt rate. This gives the spread over expected policy rates at the relevant time horizon. [3] Full details of the data series and weights used to construct this series are contained in the annex.

3.5 One issue worthy of note is that in using quoted interest rates I might not be capturing other important information about the credit environment. First, the quoted rate may not be representative of the actual rate paid by borrowers for new lending. In taking out a mortgage I might be told that I could borrow from as little as three per cent but, by the time my income and other characteristics are taken into account, I may be required to pay four per cent. Ideally I would use data about the actual cost of new borrowing but this is not available at the same level of detail as quoted rate data. Therefore I simply assume that quoted rates are representative of actual lending rates.[4]

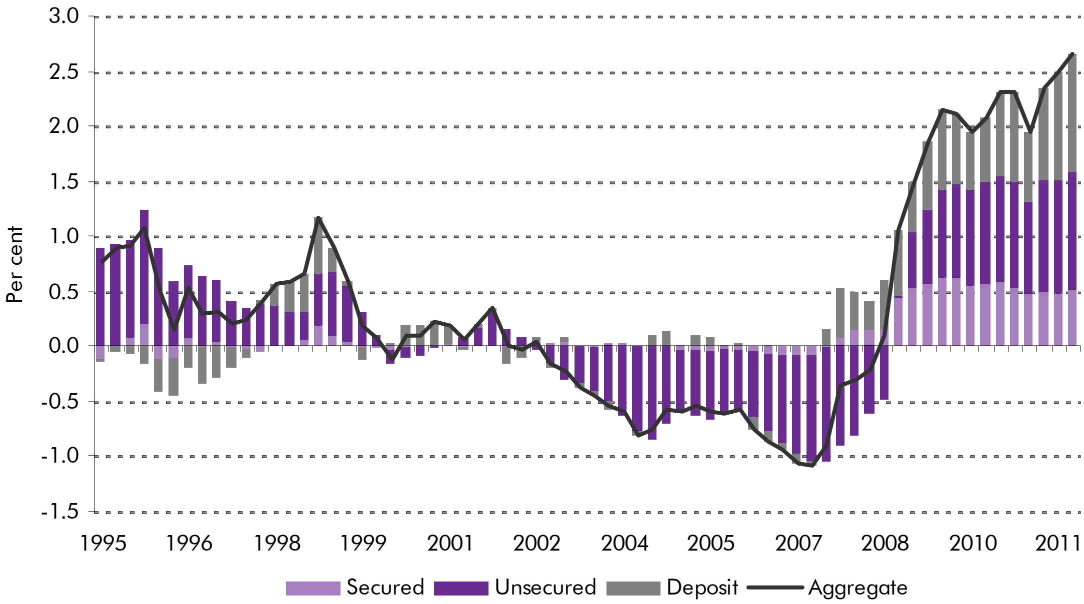
3.6 Second, while the price of lending and the return to saving are clearly important to the consumption choice, the availability of credit is also relevant. During the financial crisis, the price of lending, relative to policy rates, rose substantially but at the same time the amount of lending contracted sharply. It remains unclear whether the very slow growth in credit since the crisis represents a demand response to the weaker growth outlook and wider credit spreads or a reduced willingness to lend. In either case, the impaired functioning of the banking sector probably had significant effects on output through channels beyond those considered here or in the Curdia-Woodford paper.

3.7 In this paper, the model is presented in terms of deviations around a steady-state. Therefore, the credit spread series should also be expressed in terms of deviations around a steady-state. For simplicity, I assume that the steady-state credit spread is stationary around its long-run average value. The credit spread deviation can be decomposed into contributions from its constituent parts, which is shown in Chart 3.1.

[3] Insofar as the two-year gilt rate is a good proxy for expectations of policy rates.

[4] This is likely to be more of a problem for lending rates than deposit rates.

Chart 3.1: Contributions to the credit spread deviation from steady-state



Source: ONS, OBR

3.8 Chart 3.1 shows how the credit spread has deviated from its sample average value and the weighted contributions of secured, unsecured and deposit spreads to that deviation. Looking at the chart, it is clear that the credit spread was above its mean in the late nineties and fell below its mean for much of the following decade. An alternative explanation is that the sample mean is a poor proxy for the steady-state credit spread and that, instead, this rate has been falling over the sample period.

3.9 There are reasons to believe that the steady-state credit risk premia may have fallen over this period. There exists a substantial literature on the subject which looks at the evolution of saving behaviour on a global scale. Caballero et al. (2006) and Bernanke (2005) postulate the existence of a global savings glut which acted to reduce not just the cost of borrowing for 'safe' creditors, such as governments, but also the interest rates experienced by riskier lenders. This argument points to a structural fall in the risk premium over the sample period as global saving preferences changed.

3.10 Others, such as Taylor and Williams (2009) and Obstfeld and Rogoff (2009), are more sceptical arguing that global saving as a share of world GDP has been relatively stable over the past couple of decades and that low nominal bond yields have been a global phenomenon. Looking at Chart 3.1, if the first hypothesis were relevant to the UK experience, one might expect to see negative or falling contributions to the deviation from the sample mean from each of the three components. But, as it happens, the change is dominated by movements in unsecured borrowing spreads.[5]

3.11 Without convincing evidence in either direction, I have chosen to use the simplest specification in the model presented here, which is a constant, mean-stationary, steady-state credit spread.[6] The evolution of the credit spread is given by equation 3.1,

$$cs_t = \theta_{cs}(cs_{t-1}) + \nu_t \tag{3.1}$$

where cs is assumed to be an autoregressive process that reverts to an equilibrium mean value of zero.[7]

3.12 Following the recent establishment of the Financial Policy Committee (FPC), there may be cause to revisit this specification of the credit spread. A richer model would include a Taylor-type policy rule for the FPC, supporting the decision rule of the MPC in achieving the desired effective interest rate path.[8]

3.13 Chart 3.1 also shows that composition of the wider credit spread since the onset of the financial crisis has been evenly distributed across the three weighted lending and borrowing measures. This lends some support to the hypothesis that a widening of credit spreads is similar to a tightening of monetary policy because one would also expect higher policy rates, or expectations thereof, to feed into all three rates. If the rise in the credit spread were attributable only to deposit rates, for example, this would suggest that credit spreads were not affecting the economy in the same way as changes in conventional policy rates.

Unconventional monetary policy

3.14 As interest rates fell to the lower nominal bound in 2009, the Bank of England began to use unconventional monetary policy tools, such as the approach commonly known as quantitative easing (QE), to provide additional support to

[5] This may also reflect increased competition in the domestic unsecured lending market among banks.

[6] It is also likely that changes in regulatory policy have influenced the credit spread, an area worthy of further research.

[7] i.e. it is exogenous, as in the Curdia-Woodford model.

[8] At the time of writing, the Bank has revealed that it is considering a number of instruments with which to conduct macroprudential policy including: Those that limit the size of banks' balance sheets, those that influence the terms and conditions associated with loans and those which affect the structure of the market, Bank of England (2011).

aggregate demand.[9] In the model presented here, the Bank uses QE to achieve its targeted effective policy rate when conventional policy tools are unavailable.

3.15 The operational mechanics of QE are similar to the way in which the Bank conducts its conventional open market operations. For conventional policy, the Bank announces a target short-term interest rate and conducts open market operations – the buying and selling of short-term government debt instruments – such that the chosen interest rate is achieved. In the case of QE, the Bank announces a specific amount of longer-dated government bond purchases and allows the market to arrive at an interest rate path consistent with the size of the announced purchases.

3.16 I assume a policy rate equivalence, calibrated to the impact of QE, as estimated by Joyce et al (2011). This allows the specification of a policy rule in terms of the effective policy rate described in the credit spreads section above,

$$i_t^e = i_t + cs_t - \varphi_{qe} qe_t. \tag{3.2}$$

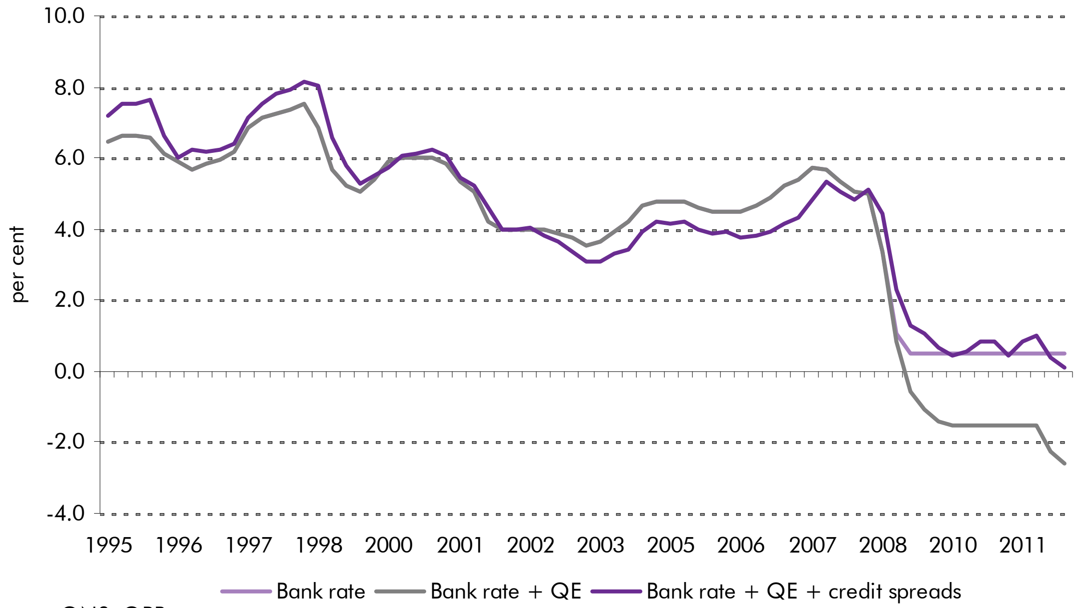
3.17 Equation 3.2 defines Effective Bank Rate, i_t^e, as a function of Bank Rate, i_t, the credit spread, cs_t and the quantum of QE in £ billion terms , qe_t, at time t. This is the interest rate that the Bank of England targets and agents in the economy respond to. The Bank is able to achieve any effective policy rate it chooses by selecting some combination of Bank Rate and Quantitative Easing policy.

3.18 One complication is that the operation of QE may affect the credit spread directly, through confidence effects. But I do not consider this channel here. That is to say that the Bank's asset purchases displace cash from gilts into other assets - boosting asset prices more generally and supporting output through net wealth effects, for example.

3.19 To show why it is important to include credit spreads and the effect of QE, Chart 3.2 illustrates three versions of the effective policy rate in the UK since 1995. The first is Bank Rate, the second is Bank Rate adjusted for the estimated impact of quantitative easing and the third is Effective Bank Rate adjusted for the estimated effect of QE and credit spreads – my preferred measure. The choice of measure leads to very different conclusions surrounding the stance of monetary policy and therefore the model's forecasts for output and inflation.

[9] And more formally known as the Asset Purchase Facility.

Chart 3.2: Different measures of the effective nominal interest rate



Source: ONS, OBR

3.20 The operation of monetary policy is characterised in this model by the assumptions presented in Table 3.1. There are seven equations in this model which capture each eventuality and are presented in the annex. The desired policy rate and actual policy rate are given by equations 3.3 and 3.4 respectively,

$$i_t^d = i_t^e - cs_t, \tag{3.3}$$

$$i_t = i_t^e - cs_t + \varphi_{qe} qe_t. \tag{3.4}$$

Table 3.1: QE policy assumptions

Desired policy rate above or below the lower bound?	Policy tightening or loosening?	Stock of outstanding QE?	QE
above	loosening	yes	no change
above	loosening	no	no change
above	tightening	yes	shrinking
above	tightening	no	no change
above	no change	yes	shrinking
above	no change	no	no change
below	loosening	yes	expanding
below	loosening	no	expanding
below	tightening	yes	shrinking
below	no change	yes	no change

The public finances

3.21 In using a small model of the economy to run simulations, it is useful to think about the possible implications for the public finances. For the purposes of producing the OBR's economic scenarios, economic determinants from a small model are used, together with off-model periphery equations, to 'ready-reckon' the effects on the public finances.[10] These are intended to roughly capture the effects of the composition of expenditure and turnover in the housing market, for example, on the public finances. But including some aggregate measures of the public finances directly in the small model presented here provides a quick and simple guide to the effect of economic simulations on the public finances without having to ready-reckon them.

3.22 This can be achieved by applying the OBR's cyclical adjustment methodology, Helgadottir et al (2012), which gives an indication of how far one might expect the cyclical component of the primary balance (net borrowing) as a share of GDP to move when the degree of spare capacity in the economy changes. i.e. how the automatic stabilisers can be expected to increase or decrease expenditure and taxation dependent on the economic cycle.

3.23 Equation 3.5 relates the cyclical component of the primary balance as a share of nominal GDP, cb, to the output gap,

$$cb_t = \xi_1 y_{t-j} \tag{3.5}$$

where ξ_1 is the cyclical adjustment parameter.

3.24 The public finances part of the model does not feed back to the other equations in any way – the effects of government debt, the structural balance or the effective gilt rate on output are not modelled here and there is no fiscal policy rule. Therefore, this part of the model can be viewed as an add-on that gives a very rough indication of the implications of the economic scenarios and simulations for the public finances.

3.25 Finally, the stock of government debt as a share of actual nominal GDP is given by the, now familiar, debt dynamics identity,

$$gdebt_t = \left(\frac{1+grate_{t-1}}{1+\Delta nY_t}\right) gdebt_{t-1} - \left(sb_t + cb_t\right). \tag{3.6}$$

[10] The OBR's approach to estimating the fiscal effects of alternative economic scenarios is presented in OBR (2012).

3.26 The stock of government debt as a share of actual nominal GDP, $gdebt$, is a function of debt in the preceding period, the effective gilt rate, $grate$, the growth rate of nominal GDP, ΔnY_t, the cyclical balance and the structural balance, sb, which is an exogenous input to the model. While running simulations, the paths of sb and $grate$ can be fixed exogenously or sourced from the central forecast, while cb, $gdebt$ and ΔnY are provided by the model presented here.[11]

3.27 It is beyond the scope of this paper to model the effect of different monetary or fiscal policy paths on the path of the effective gilt rate, which depends, among other things, on the path of policy rates, the quantum of QE and the risk premium associated with government debt. Therefore, the stylised paths the model returns should be interpreted with caution.

[11] The model here is described entirely in terms of deviations around a steady state and no attempt is made to model the supply side of the economy. However, the public finances equations described below depend on the level of actual and potential nominal GDP which, in turn, depend on the path of potential output and inflation. Potential output can be exogenously fixed on to the model but otherwise will grow at a rate consistent with its historical average.

4 Calibration and model properties

Calibration

4.1 While some models are parameterised using estimated coefficients, others are calibrated to fit certain aspects of the data. With a model this small, incomplete specification is unavoidable – there are features of recent economic history that cannot be explained within the very narrow modelling framework considered here. But this does not mean it cannot be used for the quantitative assessment of economic developments. It simply implies that accepting the estimation results presented in Annex D without some sensitivity to information that is available outside the small model would likely lead to bias. Therefore, in calibrating the model presented above, I draw from a wide range of available information. Coefficients are selected based on the empirical work of others, the estimation results presented in Annex D and, importantly, with a view to the simulation properties of the model.

4.2 As a cross-check on the empirical performance of the model I have also estimated a four-variable, two-lag VAR (of the output gap, interest rate gap exchange rate and the inflation gap[1]) and the impulse response functions are shown in Annex D. In a number of cases the VAR results provide further support for the choice of parameter size in the calibration and the lag structure of the model.

IS relation

Coefficients

4.3 In the IS relation, three coefficients: β_y, β_r and β_{er} need to be calibrated. β_y is the coefficient on the autoregressive term and can be interpreted as the speed at which the output gap can be expected to close in the absence of monetary policy action. This is determined by structural factors such as the degree of real wage flexibility in the economy, which acts as a force to restore output to its potential level. It is not simply the observed empirical persistence of the output gap and should, therefore, be characterised by a higher degree of persistence.

[1] This is also the Cholesky ordering of the variables in the impulse response functions.

4.4 Batini and Haldane (1999) select a parameter of 0.8 for the persistence of the output gap, which they consider to be empirically plausible, based on quarterly data. I choose a higher coefficient of around 0.96 reflecting the fact that the output gap series I use has a persistence of around 0.9 to begin with and the coefficient should be higher than this because it represents only the effects of real wage adjustment, not the effect of monetary policy on output. Empirical evidence of the conditional persistence of the output gap from the estimated VAR also points towards a larger coefficient.

4.5 The sensitivity of output to the real interest rate gap, β_r, is the second parameter to be calibrated in the IS relation. Nelson and Nikolov (2002) find that, from a position of equilibrium, a one percentage point increase in the real interest rate gap leads to a negative output gap in the next quarter of around 0.1 per cent. This is consistent with the effect of a one-year interest rate shock building to have a maximum effect on the output gap of roughly 0.4 percentage points.

4.6 As reported in Nelson and Nikolov, there exists a wide range of estimates of this elasticity, which is of key importance to the model, and their estimate is a little smaller than the others they discuss. But most other studies pertain to US data and there are very few empirical estimates of the slope of the IS curve for the UK. The estimation and VAR results presented in Annex D are consistent with a negative output gap of 0.05 per cent following a rise of one percentage point in the policy rate in the preceding quarter, consistent with a maximum effect on the output gap of between 0.2 and 0.3 percentage points. The output dynamics of the Bank of England's Quarterly Economic Model (BEQM) are consistent with a slightly higher coefficient.[2,3] I have chosen a parameter that is consistent with the evidence presented by Nelson and Nikolov but which falls within the 95 per cent confidence interval of the single-equation estimates presented in the annex.

4.7 Inclusion of the change in the real exchange rate within single-equation estimates of the IS relation often results in insignificant or incorrectly signed coefficients. However, the results more closely accord with theory when longer lags are included – as is the case in the OBR's main macroeconomic model. The VAR results, too, offer little support for the inclusion of the real exchange rate.

4.8 One reason for the weak empirical evidence is that the real exchange rate tends to be stable for long periods before making substantial adjustments. At these times there are likely to be other things happening that are relevant to output, such as the collapse in world trade during the recent crisis, when sterling

[2] Bank of England (2005)

[3] I use this as a reference because the simulation properties of the Bank's new model, COMPASS, are not publicly available.

depreciated around 25 per cent, or a change in the macroeconomic framework, as followed the UK's ejection from the European Exchange Rate Mechanism in the early 90s, when sterling depreciated around 15 per cent. The exchange rate could well have had a positive effect on output over these periods, ceteris paribus, but the model is not sufficiently rich to capture it – it would need to disentangle this from the negative effects associated with the global financial and exchange rate crises.

4.9 Given the uncertainties, I include a term in the IS relation for the real exchange rate but attach a relatively small coefficient to it, which is consistent with the single-equation estimates at longer lags. The effect of the real exchange rate on the output gap is just over one third as strong as the effect of the real interest rate.

Lag structure

4.10 Due to the slow adjustment of prices in the economy, a change in nominal interest rates leads to a change in real interest rates, incentivising consumption now over consumption tomorrow. It is well-known that monetary policy affects the economy with a lag and, probably, different lags at different times. This complicates the analysis of the transmission mechanism and may partly explain why the statistical relationship between real interest rates and output is often found to be so weak.

4.11 There is a wide range of estimates of the horizon over which monetary policy is effective. The estimation results are consistent with a lag of one quarter for a change in real interest rates to begin having an effect. The lag on the autoregressive component of the equation is simply one quarter. As mentioned above, I include the lagged real exchange rate in the IS relation. It is likely that firms take a long time to respond fully to the changes in relative prices that might prompt them to orientate toward or away from the export sector – particularly if there is uncertainty surrounding whether the movement in the exchange rate is permanent or temporary. Furthermore, there is little evidence that agents in the economy respond quickly to changes in relative prices, particularly with regard to substitution away from imports to domestically-produced goods. The relatively slow pass-through of the exchange rate to domestic prices may also explain some of the slow response of output to changes in the real exchange rate.

Phillips relation

Coefficients

4.12 The estimation results are consistent with an effect on the annual rate of inflation from a one percent, one-year positive output gap of around 0.4 percentage points. This is a little smaller than the coefficient presented in BEQM, but consistent with the response implied by the VAR. Groen and Mumtaz (2008) show that the Phillips curve has been quite unstable over the past few decades and others have asked whether the Phillips curve has flattened over the past few decades, such as Bean (1998).[4] I select a parameter for the slope of the Phillips curve consistent with both the estimation results and the VAR estimates but recognise that there is significant uncertainty surrounding it. I assume that the long-run effect of monetary policy on output is neutral by imposing coefficients on expected and past inflation that sum to unity.

Lag structure

4.13 Estimates of similar specifications to the hybrid NKPC used in this model have typically found that a high proportion of firms, usually around three-quarters, set prices in a forward-looking manner.[5] However, most of these papers typically side-step the problem of modelling the unobserved expectations of inflation by assuming agents have rational expectations. This assumption permits the estimation of the equation by Generalised Method of Moments (GMM) as it can now be assumed that lags of inflation are valid instruments for the expectations term.

4.14 However Rudd and Whelan (2006) provide evidence that this rational expectations assumption may not be valid. In particular, they claim that low estimates of ζ and λ_y turn out to be fully consistent with the true model being a purely backward-looking specification. Essentially, they conclude that GMM is not able to distinguish between a forward- and backward- looking representation of the Phillips curve. Given this result, and the usual finding of a good empirical fit for backward-looking specifications of the Philips curve, I therefore calibrate the model in line with the estimation and VAR results in the annex and choose a value of 0.85 for ζ , consistent with the degree of backward-looking behaviour in the rest of the model.

Taylor relation

Coefficients

4.15 A wide range of plausible estimates exist for the degree of interest rate smoothing, ψ , engaged in by central banks. A good fit is achieved by choosing

[4] For example the slope of the Phillips curve may also be a function of the level of inflation. If firms are likely to change prices less frequently when inflation is low this should lead to a weaker short term impact of demand fluctuations on prices i.e. a flatter Phillips curve.

[5] See, for example, Gali and Gertler (1999) and Batini, Nickell and Jackson (2005).

a parameter of 0.8, which ensures a degree of persistence and inertia in interest rate setting.[6]

4.16 That nominal interest rates should increase by more than inflation rises above target (so that the real interest rate rises) is known as the Taylor principle but the precise weight central banks place on inflation stabilisation over output stabilisation is the subject of an exhaustive literature. The original coefficients suggested by Taylor, and which are widely replicated, are 1.5 for β_{π} and 0.5 for β_{y} - these coefficients were obtained from empirical estimates using US data. The single, simultaneous equation and VAR estimates reported in Annex D suggest that, while the coefficient on the output gap is consistent with the Taylor rule, the responsiveness of policy rates to deviations of inflation from target is around a half. This violates the Taylor principle and, in the context of this model, would lead to an explosive inflation path.

4.17 There are a number of reasons why the estimation results might not quite get at the truth. The most obvious is that, faced with a one-off shock to the level of prices (that is unrelated to the output gap), the MPC 'looks through' this temporary influence on the rate of inflation, knowing that it will return to target without any policy intervention (provided expectations are well anchored to the inflation target). If the path for inflation has been predominantly driven by a series of shocks to the price level, then the inflation rate could exhibit a substantial amount of variability without any corresponding monetary policy response. This scenario would introduce a downward bias to the size of the estimated coefficient.

4.18 A second possibility is that the responsiveness of inflation to the output gap itself may have fallen over the estimation period. This is the so-called flattening of the Phillips curve, discussed above. That inflation has become more stable in the face of output gap shocks implies a smaller required monetary policy response.

4.19 Another way of thinking about the Taylor coefficients is in terms of what constitutes optimal monetary policy. Carlin and Soskice (2010) show that the optimal responsiveness of deviations of inflation from target can be derived from the parameters of the IS and Phillips curves,

[6] In the annex I estimate the smoothing parameter conditioned on the calibrated output and inflation coefficients. This suggests a slightly higher degree of persistence.

$$r_t - r^* = \frac{1}{\beta_r\left(\lambda_y + \frac{1}{\lambda_y \varpi}\right)}\left(\pi_t - \pi_t^*\right) \qquad (4.1)$$

where the real interest rate gap is determined by the slopes of the IS and Phillips curves and ϖ , the weight the Bank places on inflation stabilisation over output stabilisation.[7] Ultimately, it is this parameter that is most important in determining the Taylor coefficients but it is interesting to see whether the calibrated coefficients in the model could explain why the estimation results differ so vastly from the original coefficients.

4.20 Inserting the relevant parameters and assuming an equal weight on inflation and output stabilisation suggests that the Taylor coefficient on inflation would be 1.4, slightly lower than the 1.5 reported in his original paper but nowhere near the estimated coefficient.[8] This leads me to conclude that either the estimation results are correct and an assumption of unity for the ϖ term is incorrect, or that the assumption is broadly correct and the estimation results are misleading.

4.21 For the purposes of model stability, and to provide some degree of consistency in the way the Bank sets policy with the rest of the literature on the subject, the model presented here is based on the original Taylor coefficients. Clearly this is one of the most important and uncertain areas of the parameterisation process and the results that follow from using the model should be interpreted with the corresponding, and high, degree of caution.

Lag structure

4.22 The lag (or rather lead structure in this context) is determined by the laws of motion of the economy. That is, the bank has model-consistent expectations of future inflation and output and these expectations extend to the lag structure. Therefore, the lead on the output gap is one quarter and the lead on inflation is three quarters – as explained in more detail in sections 4.3 and 4.12 respectively.

[7] This is what features in the central bank's loss function.

[8] In this very simple exercise I abstract from the influence of the exchange rate, which would have a second order effect. Carlin and Soskice show how the exchange rate can be included in this approach. The coefficients inserted for this analysis are based on the maximum effect of a one-year real interest rate rise of one percentage point on the output gap and inflation, i.e. roughly 0.4 per cent and 0.4 percent respectively.

Credit spreads and unconventional monetary policy

4.23 To account for the effect of unconventional monetary policy, I use the Bank of England's estimates of the effectiveness of its Quantitative Easing (QE) program Joyce et al (2011), which equate £200 billion of QE to around a 225 basis point reduction in interest rates. This consists of an effect on the economy via lower gilt yields of around 100 basis points and further effects arising from the portfolio rebalancing channel of quantitative easing – through which higher asset prices boost output due to wealth effects, for example. This gives a β_{qe} of roughly 0.01 in terms of Bank Rate and £ billions of QE.

4.24 The speed with which QE will be unwound is unknown but, in this model, is determined by the parameter χ. The model equations imply that once the stance of monetary policy has started tightening, a program of asset sales will begin. I have calibrated this parameter based on the length of time it would take for the current quantum of QE to be unwound. This is an issue of judgement and I do not claim to have any particular insight into what the correct parameter might be. Nonetheless I choose a χ consistent with an unwinding of around £5 billion a quarter. This suggests it would take the Bank slightly under a decade to unwind the current quantum of QE.

4.25 The credit spread series is relatively persistent so I choose a β_{cs} of around 0.85 for forecasting purposes.

The public finances

4.26 Consistent with Helgadottir et al (2012), the cyclical component of the primary balance as a share of nominal actual GDP is cb. The cyclical adjustment coefficient for the current financial year is ξ_1 and the cyclical adjustment parameter for the previous financial year is ξ_2. The Helgadottir et al paper is estimated on an annual basis so the lag structure simply converts this to a quarterly basis,

$$cb_t = \xi_1\left(\frac{y_t + y_{t-1} + y_{t-2} + y_{t-3}}{4}\right) + \xi_2\left(\frac{y_{t-4} + y_{t-5} + y_{t-6} + y_{t-7}}{4}\right). \qquad (4.2)$$

4.27 This is the only core equation in this section, with the debt dynamics equation determined by an identity and some exogenous assumptions, specified in section 3.25.

Overview of calibrated equations with relevant lag structure

Core equations

$$y_t = \beta_y y_{t-1} + \beta_r (z_{t-1}) + \beta_{er} \Delta er_{t-2} + \mu_t$$

$$\pi_t = \varsigma \pi_{t-1} + (1-\varsigma)\pi^* + \lambda_y y_{t-2} + \lambda_e \Delta e_{t-3} + \varepsilon_t$$

$$i_t^e = (1-\psi)(\bar{i}_t) + (1-\psi)\gamma_y y_{t+1_{|t}} + (1-\psi)\gamma_\pi \left(\pi_{t+3_{|t}} - \pi^*\right) + \psi i_{t-1}^e$$

$$cb_t = \xi_1 \left(\frac{y_t + y_{t-1} + y_{t-2} + y_{t-3}}{4} \right) + \xi_2 \left(\frac{y_{t-4} + y_{t-5} + y_{t-6} + y_{t-7}}{4} \right)$$

Exogenous equations

$$Y_t^* = \tau_{Y*}(Y_{t-1}^*)$$

$$cs_t = \theta_{cs}(cs_{t-1})$$

$$i_t = i_t^e - cs_t + \varphi_{qe} qe_t$$

Table 4.1: Calibrated coefficients

Parameters	Coefficients
β_y	0.96
β_r	-0.1
β_{er}	0.04
ς	0.85
λ_y	0.1
λ_e	-0.06
ψ	0.8
γ_y	0.5
γ_π	1.5
φ	0.01
θ	0.85
ξ_1	0.5
ξ_2	0.2
τ_{Y*}	1.0057
χ	5

Simulation properties

Demand shock

4.28 The response of the model to a demand shock is consistent with the consensus view of the operation and responsiveness of monetary policy and the transmission mechanism. Following a 1 per cent negative shock to the level of output, sustained for four quarters, annual inflation falls around ½ a percentage point below target after two years, before gradually being brought back to target by the monetary policy response. The nominal Effective Bank Rate falls by more than 1 per cent in four quarters, producing a fall in real interest rates that returns output to its steady-state (potential) level. Government debt as a share of nominal GDP rises due to additional cyclical spending and lower GDP. The lower rate of GDP inflation also has a permanent denominator effect on the debt ratio as the price level (and so nominal GDP) remains lower than would otherwise have been the case.

Interest rate shock

4.29 A one-year, one percentage point increase in nominal interest rates leads to a negative output gap which widens to around 0.3 per cent of potential output after a year. The wider margin of spare capacity causes inflation to fall, such that the deviation from target builds to around 0.1 percentage points after two years. The real exchange rate initially depreciates before slowly appreciating as the interest rate differential with the rest of the world closes. Again government debt is a little higher reflecting additional cyclical spending and lower nominal GDP.

Nominal exchange rate shock

4.30 A 10 percent appreciation of sterling has a direct, albeit relatively small, effect on the level of output of around 0.2 per cent, which reaches its maximum around one year after the shock begins. The effect on inflation is a little larger, causing a deviation of inflation from target of around -½ a percentage point after one year – reflecting the slow pass-through of movements in the exchange rate to the price of domestically-produced output. The lower inflation and weaker output triggers a policy response, with a lower Effective Bank Rate aimed at returning inflation to target and output to its potential level. Government debt is a little higher at the end of the simulation period reflecting additional cyclical spending and lower nominal GDP.

Inflation shock

4.31 In this model, an inflation shock can be interpreted as a cost-push shock of domestic origin – such as a higher degree of wage-bargaining power on the part of workers or a push for wider margins by firms. A positive one per cent inflation shock, sustained for one year provokes a monetary tightening which results in weaker output of around 0.2 per cent after a year and a half. The real exchange rate depreciates as the depreciation of sterling more than outweighs the effect of the inflation differential between the UK and its trading partners. Government debt is a little lower, as a share of nominal GDP, by the end of the simulation period due to the effect of higher domestically-generated inflation.

Determinacy

4.32 It is important to assess whether the system of equations presented in the earlier sections is consistent with generating a uniquely-identified equilibrium which is not explosive.[9] It is relatively clear from the simulations presented below that, when presented with a number of shocks, the system of equations delivers a return to equilibrium that is consistent with the steady state properties of the model. Similarly, that the model solves shows that a unique equilibrium exists. Yet it is helpful to think about this analytically.

4.33 Whether a model suffers from indeterminacy depends on the number of 'jump' or forward-looking variables and the persistence associated with each of the variables in the system of equations – i.e. how many contain a random walk component. The only variable which is determined by information which only becomes available in the current period is the exchange rate.[10] The exchange rate is also the only variable which contains a random walk element - the coefficient on the lagged exchange rate is equal to unity. The Taylor rule does have a forward-looking component but, because the Bank's expectations are model-consistent and the other equations are predetermined, it too is predetermined.

4.34 The model can be considered determinate if the number of jump variables is exceeded by the number of variables which do not contain a random walk component. We know that there is one jump variable – the exchange rate – and that there are three variables which do not contain a random walk component. As we know that the number of equations without a random walk component is greater than the number of equations that are non-predetermined we can safely conclude that this model has a determinate solution.

[9] Walsh (2003) explains some of the problems associated with indeterminacy. The usual cause is a central bank reaction function which violates the Taylor principle – this leads to an explosive path for inflation, the output gap and the exchange rate.

[10] The exchange rate is contemporaneously determined by the interest rate

5 Fiscal policy announcements and interest rate forecasts

5.1 Around the time of the June 2010 Budget, the interim OBR produced two economic forecasts. The first, published ahead of the June Budget, represented an updated economic forecast based on the fiscal plans set out by the previous Government in its March 2010 Budget forecast (the 'pre-measures' forecast). The second, published alongside the June Budget, was an economic forecast based on the new Government's fiscal plans (the 'post-measures' forecast).

5.2 The OBR's macroeconomic forecasts are conditioned on the path of interest rates expected by financial market participants, derived from money market and bond instruments. At the time the pre-measures forecast was put together, it was recognised that market participants expected the new Government to announce in the upcoming Budget a much larger fiscal consolidation than in the previous Government's plans. Therefore, the path of market interest rates was likely to be inconsistent with the fiscal plans on which the pre-measures forecast was based. Specifically, interest rate expectations were likely to have been lower than if they had been conditioned on the previous Government's plans. As the OBR explained at the time this may have introduced an upward bias to the GDP profile in the pre-measures forecast.[1]

5.3 In the post-measures forecast, the interim OBR decided to use the same interest rate path as in the pre-measures forecast. It explained at the time that this meant that comparing the pre- and post-measures forecasts might not be a firm basis for estimating the impact on the economy of the new Government's fiscal consolidation, but that the degree of any bias was very difficult to calculate.

5.4 There are very few types of aggregate demand shock for which the timing and size are known with any certainty and few models are set up to deal with expected aggregate demand disturbances such as pre-announced fiscal-policy measures. The model developed here can be extended to provide some indication as to the potential magnitude of the change in interest rate expectations for a given amount of additional fiscal tightening or loosening. Of

[1] See OBR (2010), Box C.1, page 94 for further explanation.

course, this model is highly stylised and the results should be interpreted as a broad guide rather than a precisely-calibrated estimate.

5.5 The baseline IS relation presented in the annex can be extended to include a fiscal policy shock, f_{t}, expressed in terms of aggregate demand,

$$\mathrm{y}_{\mathrm{t}} = \beta_{\mathrm{y}} \mathrm{y}_{\mathrm{t-1}} + \beta_{\mathrm{r}} (\mathrm{z}_{\mathrm{t-1}}) + \beta_{\mathrm{er}} \Delta er_{t-2} + \mathrm{f}_{\mathrm{t}} + \mu_{\mathrm{t}}. \tag{5.1}$$

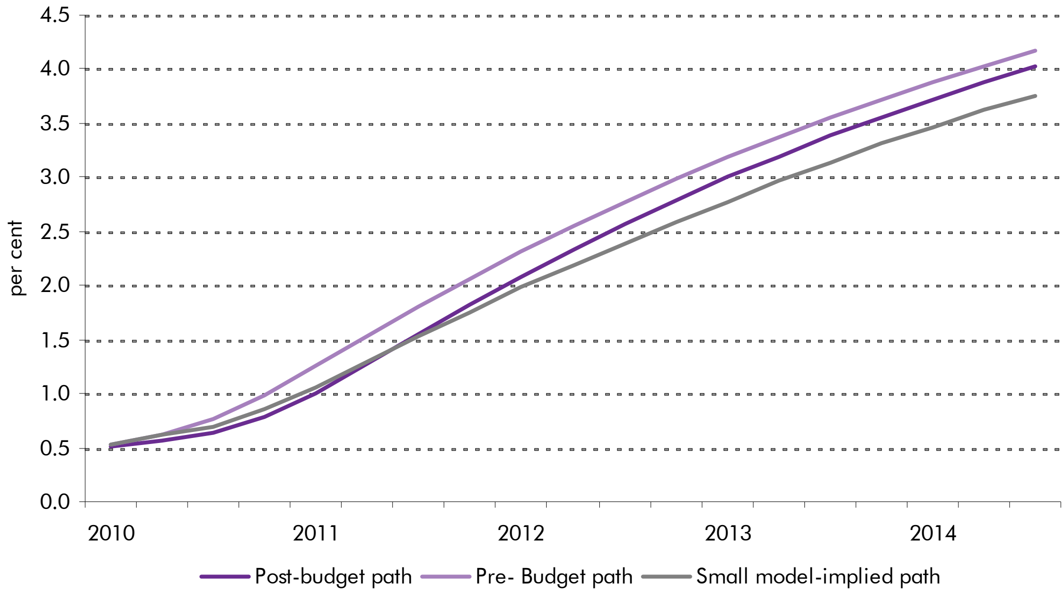
5.6 In the case of the fiscal policy adjustment announced in the June Budget, the direct output effect of the additional tightening is given by the product of the size of the additional tightening and the relevant impact multipliers. The sum of those spending and taxation measures is also equal to the change in the term, sb, discussed in para 3.26. Taking these as a share of potential nominal output gives the direct effect of the additional tightening in output gap space and should be thought of as the effect of the measures before any offset from monetary policy or real wage adjustment.

5.7 The Taylor rule presented in Chapter 3 is unchanged by the introduction of the policy shock but the Bank responds via its expectation of the output gap, which now takes announced fiscal adjustments into account. This implies that the Bank reacts in advance to fiscal policy announcements and sets interest rates with a view to stabilising output and inflation. It is worth noting that in the baseline model it takes interest rates three quarters to affect inflation so three-quarter-ahead inflation is what features in the Taylor rule (it is the earliest that the Bank can hope to have an influence on the target variable). If the Government were to announce additional tightening or loosening five years ahead, it seems reasonable that the Bank might act sooner than is implied by the Taylor rule specified in this simple baseline model.

5.8 The small model implies that, taking the additional fiscal tightening announced at the time of the June 2010 Budget into account, interest rates would be lower when compared with a baseline of unchanged fiscal policy. This is illustrated in Chart 5.1. The Pre-Budget series is the interest rate path, derived from financial market instruments, on the day of the June 2010 Pre-Budget forecast. The Post-Budget series is the interest rate path on the day of the June 2010 Budget forecast. The small model-implied series is the Pre-Budget path, adjusted for the monetary policy response implied by the model.

5.9 At the time of the June 2010 Budget only the Pre-Budget interest rate path was available to the interim OBR. This is likely to have contained, already, some expectation of a tightening of fiscal policy over the forecast period. The small model projection can, then, be thought of as the path of interest rates one might have expected to see if the additional tightening was completely unexpected by financial market participants.

Chart 5.1: Interest rates before and after the June 2010 Budget



Source: ONS, OBR

5.10 After the Budget was announced, financial market participants revised-down their expectations of interest rates and this would have been the most appropriate path of interest rates upon which to condition the post-measures forecast. Because we can never know the degree to which policy is expected, it is impossible to predict exactly where the post-measures line will fall. But the small model can be used to provide some indication as to the range of possible interest rate paths one might expect for a given set of policy announcements.

5.11 Chart 5.1 implies that, at the time of the June 2010 Budget, financial market participants were not expecting any tightening in the short term (or perhaps even a very small loosening), and that the cuts would be back-loaded. It also suggests, that while some consolidation was expected in the medium term, market participants were surprised by the size of the announced tightening – with a bit over half the total priced-in towards the end of the forecast period.

5.12 It is worth highlighting the considerable uncertainties surrounding this assessment. First, one must assume that financial market participants agree that the economy works in the way the model describes, from the willingness of the Bank to smooth output volatility right down to the response of inflation to the output gap. Second, market participants would also have to agree with the impact multipliers used to calculate the direct effect of the policy measures in output gap space. Third, the assumption is made that market participants believe that the policy announced will actually be implemented.

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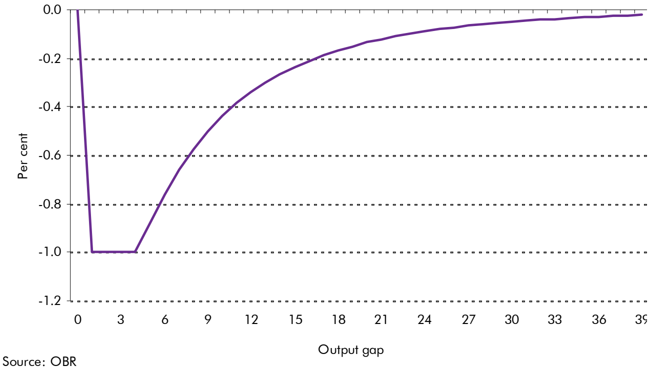
B Simulation charts

Demand shock



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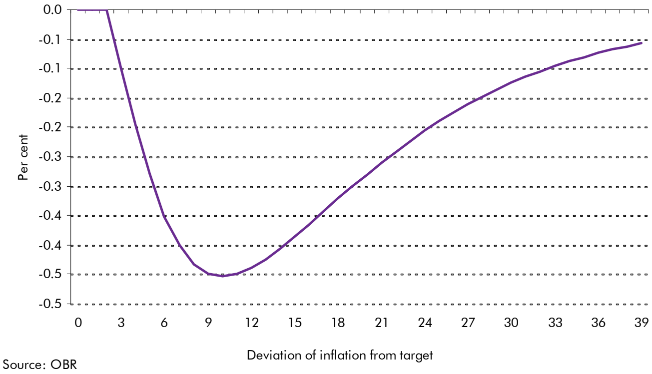

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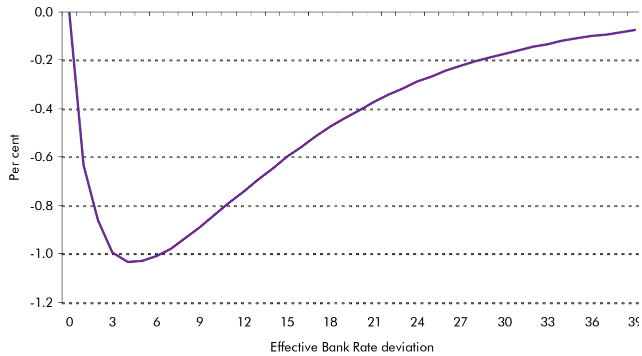

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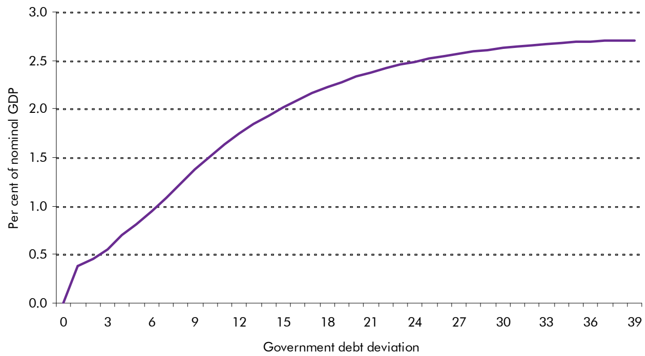

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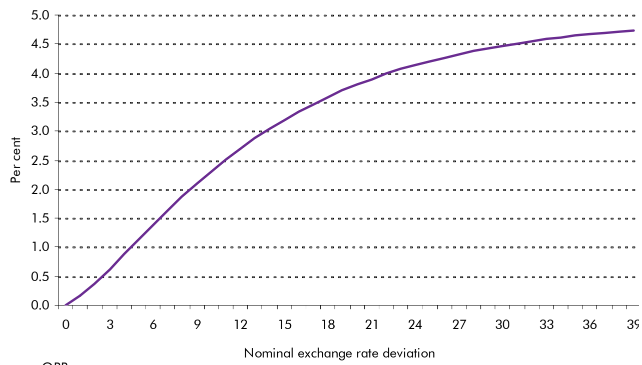

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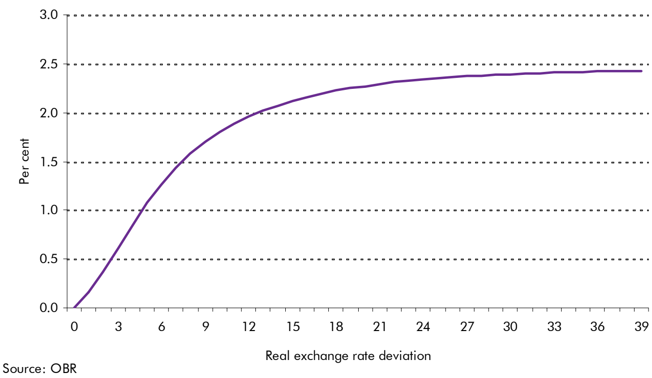
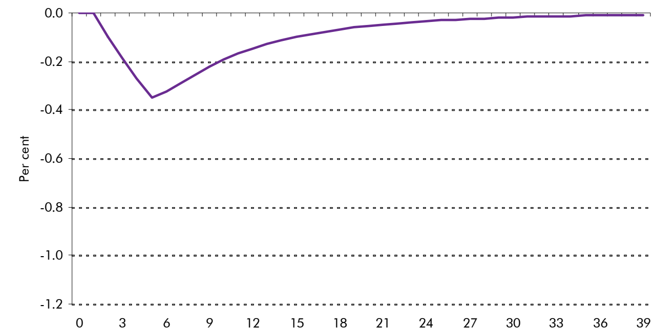


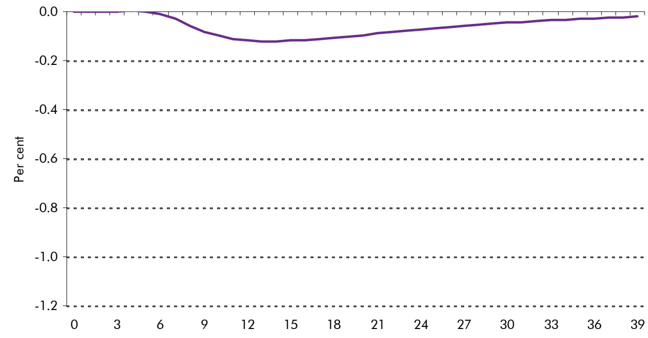
Source: OBR

Interest rate shock



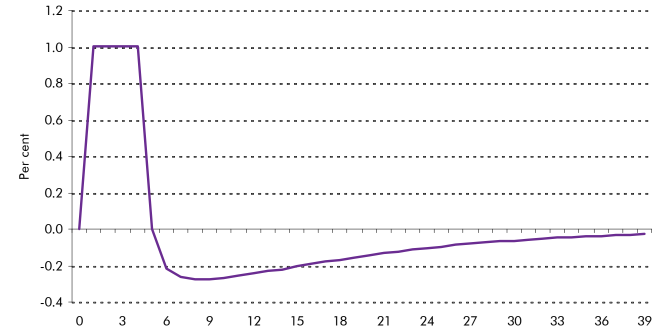
Output gap

Source: OBR



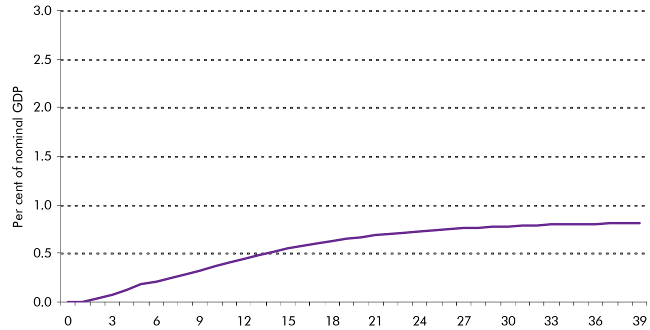
Deviation of inflation from target

Source: OBR



Effective Bank Rate deviation

Source: OBR

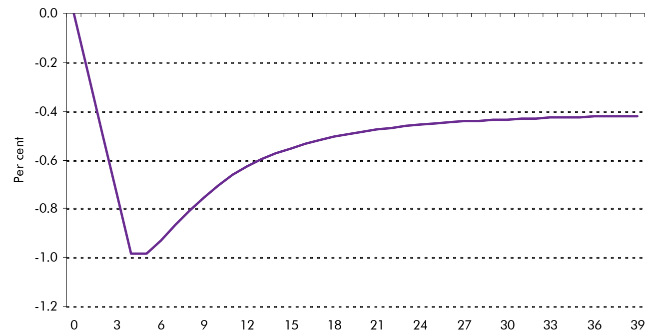


Government debt deviation

Source: OBR

Nominal exchange rate deviation

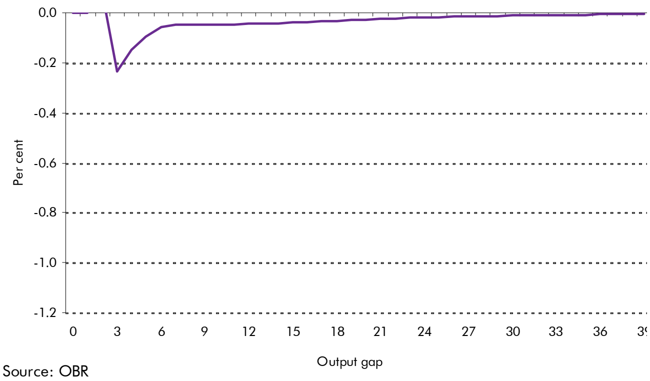
Source: OBR



Real exchange rate deviation

Source: OBR

Exchange rate shock



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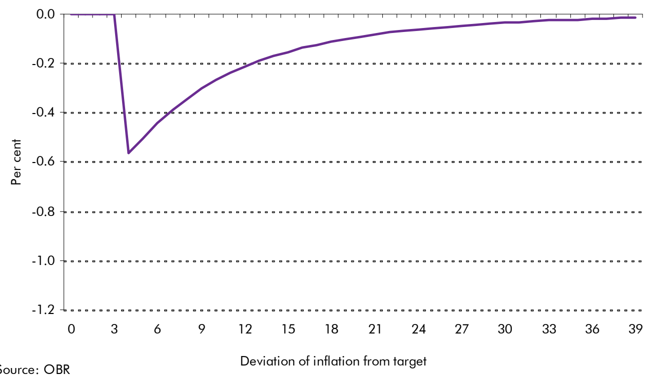

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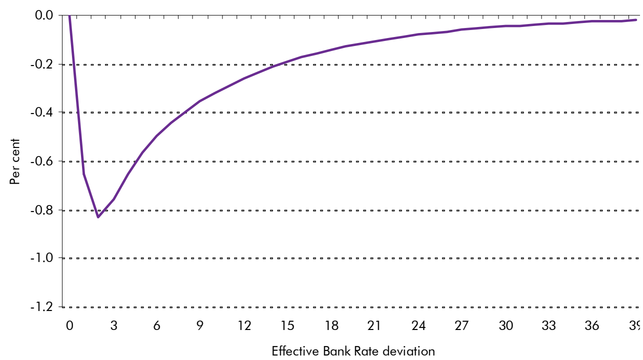

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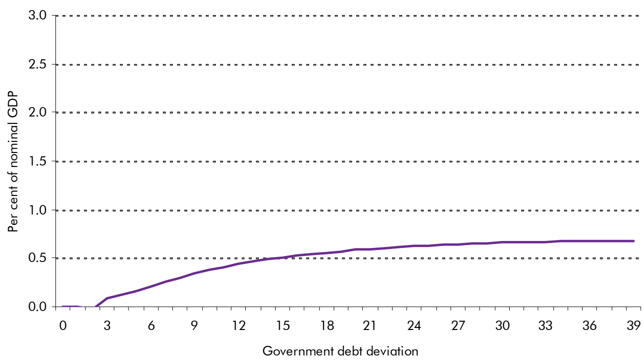

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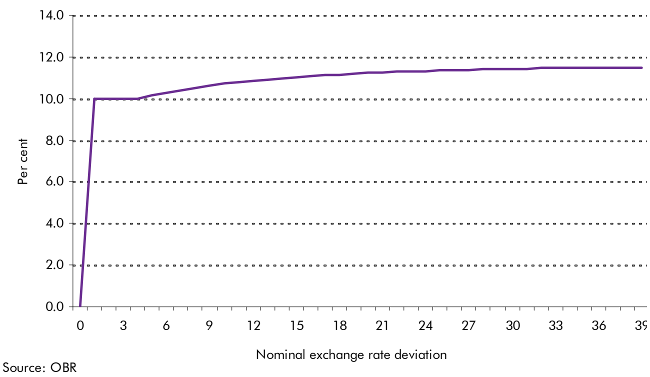

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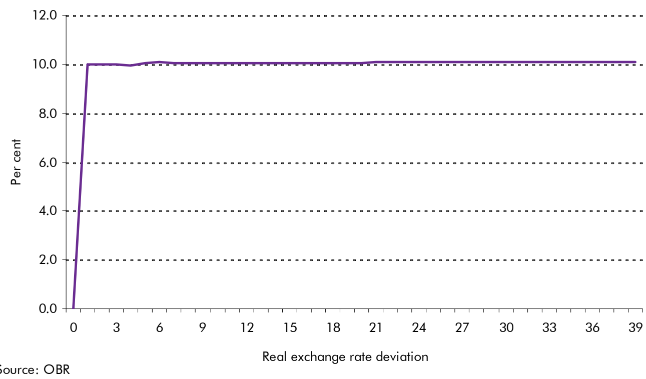
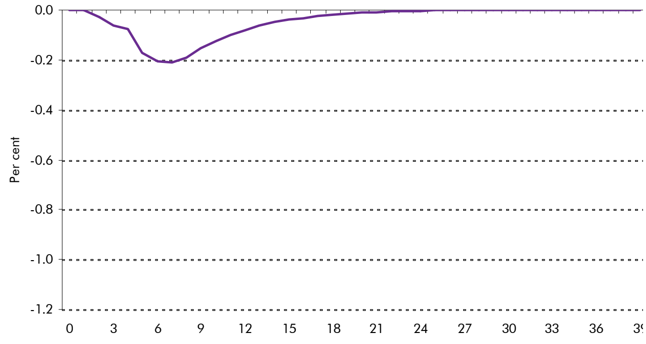


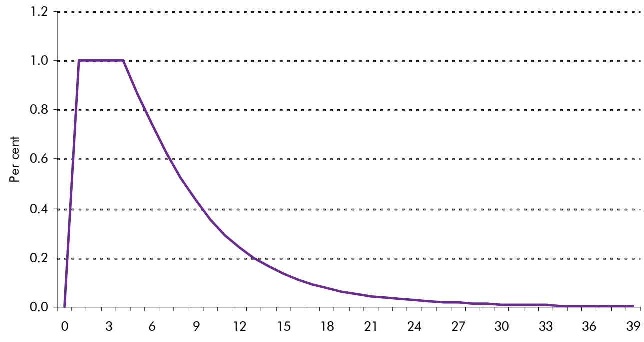
Source: OBR

Inflation shock



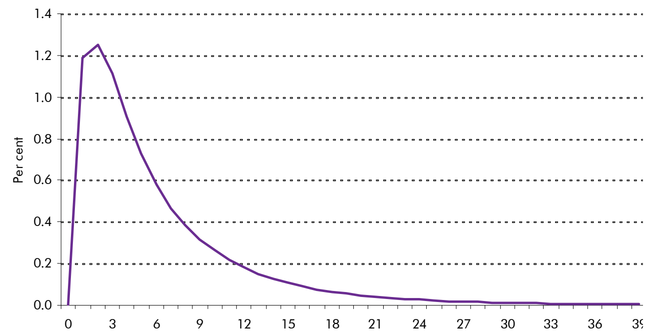
Output gap

Source: OBR



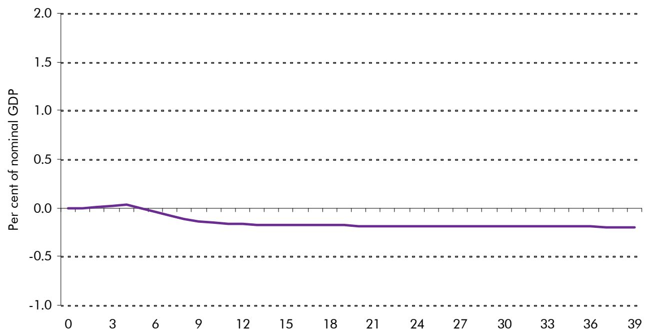
Deviation of inflation from target

Source: OBR



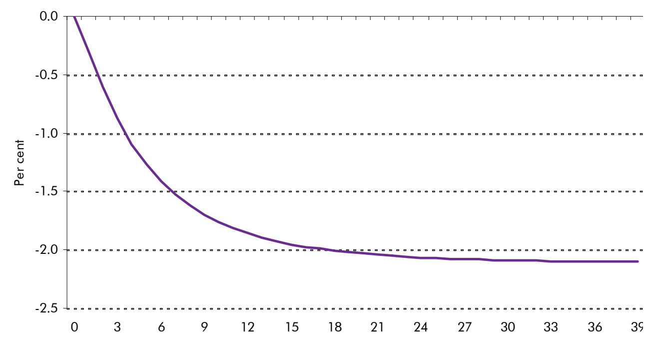
Effective Bank Rate deviation

Source: OBR



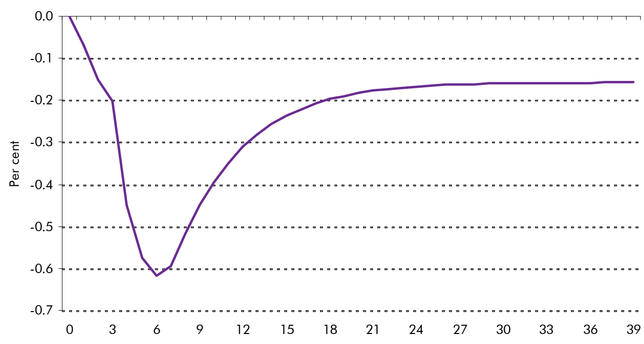
Government debt deviation

Source: OBR



Nominal exchange rate deviation

Source: OBR



Real exchange rate deviation

Source: OBR

C Model equations and data

Model equations

Core equations

IS relation

$$y_t = \beta_y y_{t-1} + \beta_r (z_{t-1}) + \beta_{er} \Delta er_{t-2} + \mu_t \quad (C.1)$$

Phillips curve[1]

$$\pi_t = \varsigma \pi_{t-1} + (1-\varsigma)\pi^* + \lambda_y y_{t-2} + \lambda_e \Delta e_{t-3} + \varepsilon_t \quad (C.2)$$

Taylor rule

$$i_t^e = (1-\psi)(\bar{i}_t) + (1-\psi)\gamma_y y_{t+1_{|t}} + (1-\psi)\gamma_\pi \left(\pi_{t+3_{|t}} - \pi^*\right) + \psi i_{t-1}^e \quad (C.3)$$

Uncovered interest parity

$$e_t = e_{t-1} + i_t^f - i_t + \wp_t \quad (C.4)$$

Cyclical budget balance

$$cb_t = \xi_1 \left(\frac{y_t + y_{t-1} + y_{t-2} + y_{t-3}}{4} \right) + \xi_2 \left(\frac{y_{t-4} + y_{t-5} + y_{t-6} + y_{t-7}}{4} \right) \quad (C.5)$$

Exogenous equations

Potential output

$$Y_t^* = \tau_{Y*}(Y_{t-1}^*) \quad (C.6)$$

Credit spread

$$cs_t = \theta_{cs}(cs_{t-1}) \quad (C.7)$$

[1] In the actual model code, this is expressed in terms of the deviation of inflation from target

Foreign interest rates

$$i_t^f = i_{t-1}^f \tag{C.8}$$

Foreign inflation

$$\pi_t^f = \pi_{t-1}^f \tag{C.9}$$

Gilt rate

$$grate_t = grate_{t-1} \tag{C.10}$$

Steady-state real interest rate

$$r_t^* = r_{t-1}^* \tag{C.11}$$

Structural balance

$$sb_t = sb_{t-1} \tag{C.12}$$

Identities

Debt to nominal GDP ratio

$$gdebt_t = \left(\frac{1 + grate_{t-1}}{1 + d.nY_t}\right) gdebt_{t-1} - \left(sb_t + cb_t\right) \tag{C.13}$$

Real GDP

$$Y = Y_t^* + (Y_t^* (y_t / 100)) \tag{C.14}$$

Nominal GDP

$$nY_t = Y_t \bullet PY_t \tag{C.15}$$

GDP deflator

$$PY_t = PY_{t-1} \bullet (1 + \pi_t) \tag{C.16}$$

Real interest rate gap

$$z_t = r_t - r_t^* \tag{C.17}$$

Real interest rate

$$r_t = i_t^e - \pi_t \tag{C.18}$$

Real exchange rate

$$\Delta er_t = (e_t - e_{t-1}) + \pi_t - \pi_t^f \tag{C.19}$$

Actual Bank Rate

$$i_t = i_t^e - cs_t + \varphi_{qe} qe_t \tag{C.20}$$

Quantitative easing

When $i_t^d < lb_t$ and $i_t^d - i_{t-1}^d < 0$, then:

$$qe_t = qe_{t-1} + \left(\frac{lb_t - i_t^d}{\varphi}\right) - \left(\frac{lb_{t-1} - i_{t-1}^d}{\varphi}\right) \tag{C.21}$$

When $i_t^d - i_{t-1}^d > 0$ and $qe_{t-1} > 0$, then:

$$qe_t = qe_{t-1} - \chi \tag{C.22}$$

When $i_t^d > lb_t$, $i_t^d - i_{t-1}^d = 0$ and $qe_{t-1} > 0$, then:

$$qe_t = qe_{t-1} - \chi \tag{C.23}$$

When $i_t^d - i_{t-1}^d > 0$ and $qe_{t-1} = 0$, then:

$$qe_t = 0 \tag{C.24}$$

When $i_t^d - i_{t-1}^d = 0$ and $qe_{t-1} = 0$, then:

$$qe_t = 0 \tag{C.25}$$

When $i_t^d > lb_t$ and $i_t^d - i_{t-1}^d < 0$, then:

$$qe_t = qe_{t-1} \tag{C.26}$$

When $i_t^d < lb_t$, $i_t^d - i_{t-1}^d = 0$ and $qe_{t-1} > 0$, then:

$$qe_t = qe_{t-1} \tag{C.27}$$

5.13 There are seven equations which govern the operation of QE in the model. The first (C.21) simply says that when the desired nominal policy rate is below the lower bound and the movement in policy is expansionary, the Bank expands its asset purchases. Where i_t^d is the desired nominal policy rate and lb_t is the lower bound at which conventional monetary policy is thought to be effective.

C.1 The second equation (C.22) is rather more difficult because there has not been any specific guidance provided by the Bank of England that explains how QE will be unwound. It seems likely that, when policy tightening begins, interest rates will rise before the stock of assets that have been purchased by the Bank begin to be sold. However, this would be complicated to introduce to a small model so I simply assume that, if there is an outstanding quantum of QE, as policy tightens, the Bank increases interest rates and sells assets at the same time. The speed at which QE is unwound is given by the parameter χ.

C.2 The third equation (C.23) says that when the desired policy rate is above the lower bound and the stance of policy is unchanged (i.e. i_t^e is at its steady-state value) but there is an outstanding stock of QE, the Bank unwinds this stock at a speed determined by χ.

C.3 The fourth equation (C.24) says that when policy is tightening and the stock of QE is zero, then it remains zero.

C.4 The fifth equation (C.25) says that when the policy stance is unchanged and the stock of QE is zero, then it remains zero.

C.5 The sixth equation (C.26) simply states that when the desired policy rate is above the lover bound and policy is loosening, the Bank moves nominal interest rates, rather than purchasing assets.

C.6 And, finally, the seventh equation (C.27) says that when the desired policy rate is below the lower bound the policy stance is unchanged and there is an outstanding stock of QE, then there is no change in QE.

Model data

IS relation

C.7 Equation D.1 is the IS relation, where y_t is the OBR's estimate of the historical output gap, Pybus (2011), z_{t-j} is the real interest rate gap.

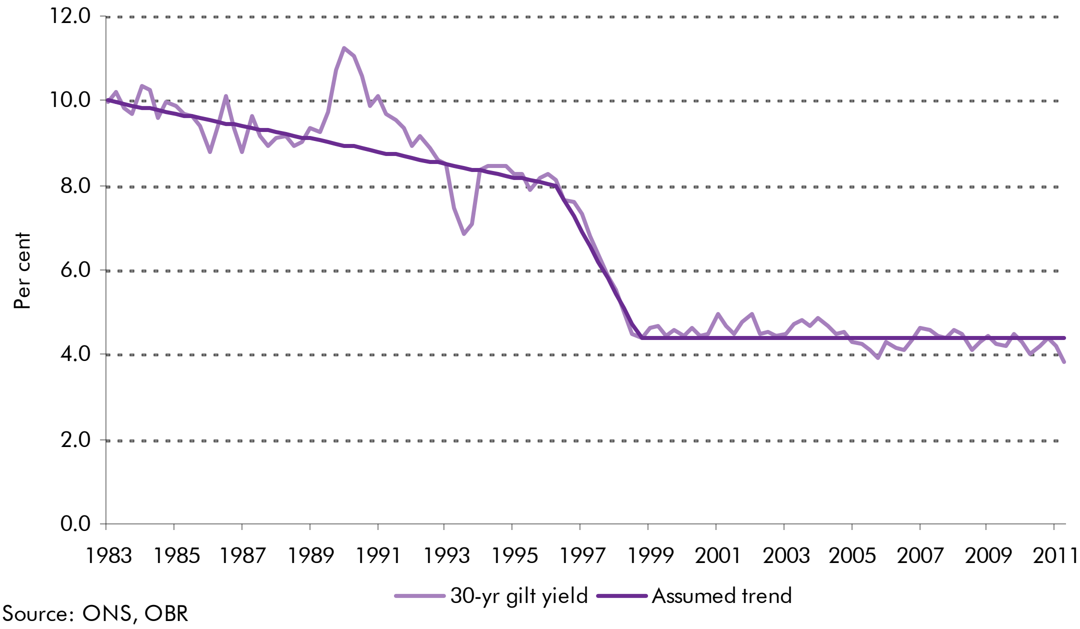
C.8 Defining the real interest rate gap is of critical importance to the estimation and performance of the model. equation C.28 sets out the Fisher relation in interest rate gap space,

$$z_t = (i_t^e - \bar{i}_t^e) - (\pi_t - \pi_t^*). \tag{C.28}$$

C.9 The real interest rate gap is equal to the nominal interest rate gap less the inflation gap. Where i_t^e is Effective Bank Rate, $\bar{i}_t^e$ is the equilibrium bank rate and $(\pi_t - \pi_t^*)$ is the deviation of inflation from target. Inflation is defined as annual GDP deflator growth at factor cost. I choose this particular measure because it abstracts from temporary influences on inflation from indirect taxation measures, such as Value Added Tax – which has had a significant effect on headline inflation in the past few years. It is also a measure of domestically-generated inflation, based on the idea that changes in import prices, for example, are only explicitly taken into account to the extent that they feed into the price of domestic output. The inflation target has been set at 2.6 per cent, the annual GDP deflator growth rate believed to be consistent with annual Consumer Prices Index inflation of two per cent in the long run.

C.10 The equilibrium nominal interest rate presents something of a challenge. It is clear from a cursory examination of the Bank Rate series that there has been a structural fall in this variable. As I state in section x.x, this is unlikely to be driven entirely by changes in the credit risk premium and for the purposes of constructing this model, I assume that the equilibrium real rate of interest is constant. Instead, it is likely to reflect changes in long-run expectations of inflation and the inflation risk premium itself. To allow for a falling equilibrium nominal interest rate I use the change in the 30-year nominal gilt rate as a proxy for the evolution of the inflation risk premium and adjust the steady-state nominal interest rate series accordingly.[2]

Chart C.1: Assumed trend of nominal steady-state interest rate



Source: ONS, OBR

C.11 Chart C.1 shows the 30-year gilt rate and its assumed trend, used to adjust the steady-state nominal interest rate. It is not until the Bank of England is granted independence that the inflation risk premium begins to fall sharply, before stabilising. The period of elevated rates in 1990 reflects attempts to remain in the European Exchange Rate Mechanism and the subsequent dip, the recession that followed. I use the assumed trend to adjust the nominal interest rate gap, which features in the fisher relation, and therefore the real interest rate gap. This ensures the stationarity of the real interest rate gap series used to estimate the IS relation.

C.12 For the purposes of forecasting, I assume a constant equilibrium nominal interest rate equal to the average effective policy rate over a period in which the output gap is thought to have been zero on average.[3] This is supported by the relative stability of the 30-year gilt rate in the years following Bank of England independence – inflation expectations and the inflation risk premium are assumed to be well-anchored in the forecast period.

[2] I have selected the period 1999Q1-2011Q3.

Phillips curve

C.13 In equation C.2, π_t is the growth of the GDP deflator at factor cost – the choice of this measure of inflation is discussed in para C.22. y_t is the OBR's estimate of the historical output gap and e_t is the nominal sterling effective exchange rate, which is trade-weighted.

Exchange rate

C.14 In equation C.4, the nominal exchange rate, e, is the effective exchange rate, as published by the Bank of England. The foreign interest rate, i^f, is exogenous to the model but is essentially the counterpart the domestic nominal policy rate set by the Bank of England, i. The change in the log real exchange rate – equation C.19 - is given by the domestic-foreign inflation differential. The inflation measures used are the UK GDP deflator at factor cost and the OECD-compiled world consumption deflator.[4]

Taylor rule

C.15 Equation C.3 is the Taylor rule, as described in section x. Where i_t^e is Effective Bank Rate, set by the Bank of England, y is the OBR estimate of the historical output gap, π is quarterly annualised GDP deflator growth at factor cost and π^* is the inflation target – assumed to be 2.6 per cent - the long-run average of GDP deflator growth at factor cost assumed to be consistent with CPI inflation at the target rate of 2 per cent.

[4] There seems to be little difference between world consumption and domestic GDP deflator long-run average rates. In the forecast the two are constrained to be equal in steady state to ensure stability of the real exchange rate in steady state.

Table C.1: Model data table

Variable	Description	Source	Identifier
y	The output gap (non-oil GVA basis)	OBR calcs. - Pybus (2011)	N/A
i	Bank Rate	BoE	
qe	Quantitative easing (£bn)	BoE	N/A
cs	Credit spread	OBR calcs. – Table 5.2	N/A
π	GDP at factor cost deflator growth	OBR calcs./ONS	N/A
π^f	OECD consumption deflator growth	OECD	N/A
e	Nominal effective Sterling exchange rate	BoE	XUQABK67
i^f	Trade-weighted average of short-term interest rates for major economies	OBR calcs.	N/A
Y	Real GDP	ONS	ABMI
nY	Nominal GDP	ONS	YBHA
Y^*	Potential non-oil GVA	OBR	N/A
P	GDP deflator	ONS	YBHA/ABMI
$gdebt$	Government debt as a percentage of nominal GDP	ONS	HF6X
$grate$	Effective gilt rate	OBR calcs.	N/A

Table C.2: Credit spread data definitions and weights

Rate series	Bankstat I.D.	Weight	Reference rate	Bankstat I.D./data source
75% LTV - Bank rate tracker	IUMBV24	0.09	Bank rate	IUQABEDR
75% LTV - 2 yr fixed	IUMBV34	0.08	2yr gilt	REUTERS
75% LTV - 5 yr fixed	IUMBV42	0.04	5yr gilt	REUTERS
90% LTV - 2 yr fixed	IUMB482	0.02	2yr gilt	REUTERS
Credit card	IUMCCTL	0.12	Bank rate	IUQABEDR
£10k personal loan	IUMHPTL	0.06	Bank rate, 2yr gilts, 5 yr gilts	IUQABEDR, REUTERS
£5k personal loan	IUMBX67	0.06	Bank rate, 2yr gilts, 5 yr gilts	IUQABEDR, REUTERS
Overdraft	IUMODTL	0.03	Bank rate	IUQABEDR
Time deposit	IUMWTTA	0.1	Bank rate	IUQABEDR
Fixed bond rates	IUMWTFA	0.4	1yr gilt, 2yr gilts, 3yr gilts, 5yr gilts	REUTERS

D Estimation results

Single-equation ordinary least squares

D.1 The results presented in Table D.1 are from single-equation ordinary least squares estimation of the three core behavioural equations – the IS, Phillips and Taylor relations. The estimation period is from the first quarter of 1982 and the third quarter of 2011. Stationarity tests reveal that a null hypothesis of stationarity cannot be rejected for each of the model variables, with the curious exception of the output gap – which is stationary by construction.

Table D.1: Single-equation estimates

Parameters	Coefficients	95% conf interval	Significance[1]
β_y	0.96	0.95 – 1.06	*
β_r	-0.05	-0.10 – -0.00	*
β_{er}	-0.03	-0.06 – 0.00	***
ς	0.88	0.81 0.96	*
λ_y	0.12	0.01 – 0.23	**
λ_e	-0.06	-0.12 – 0.00	***
ψ [2]	0.9	0.86 – 0.98	*
γ_y [3]	0.51	034. – 0.69	*
γ_π	0.62	0.45 – 0.78	*

[1] * indicates significance at 1 per cent, ** at 5 and *** at 10.

[2] Smoothing parameter extracted by constraining Taylor coefficients to 0.5 and 1.5 for the output gap and inflation deviation from target respectively

[3] Some evidence of heteroskedasticity in the residuals of the single-equation estimate of the Taylor rule – suggests bias

Three-stage ordinary least squares

D.2 Table D.2 presents the results of the three-stage least squares estimation of the model over the same estimation period as the single-equation estimates presented above. The IS and Phillips relations were estimated jointly but I dropped the Taylor rule from the model at this stage, reflecting concerns over bias in the estimated coefficients.

Table D.2: Three-stage least squares estimates

Parameters	Coefficients	95% conf interval	Significance[1]
β_y	1.00	0.95 – 1.05	*
β_r	-0.05	-0.10 – 0.01	**
ς	0.87	0.79 – 0.95	*
λ_y	0.13	0.02 – 0.24	**
λ_e	-0.06	-0.13 – 0.00	**

[1] * indicates significance at 1 per cent, ** at 5 and *** at 10.

Vector autoregression (VAR) results

D.3 The results for estimating a four-variable, two-lag VAR that includes the output gap, inflation gap, real interest rate gap and the nominal exchange rate variables, used in the small model, are shown in Table D.3. The estimation period is the first quarter of 1982 to the third quarter of 2011. Of more interest are the VAR's impulse response functions shown in the charts and discussed in the calibration section of the paper.

Table D.3: VAR sum of coefficient matrix

Variable	y	z	Δe	$\pi_t - \pi^*$
y	0.98	0.11	-0.09	0.12
z	-0.04	0.84	0.08	-0.01
Δe	0.00	0.05	-0.07	-0.05
$\pi_t - \pi^*$	-0.02	0.03	0.12	0.89

Chart D.1: Response of output gap to shocks

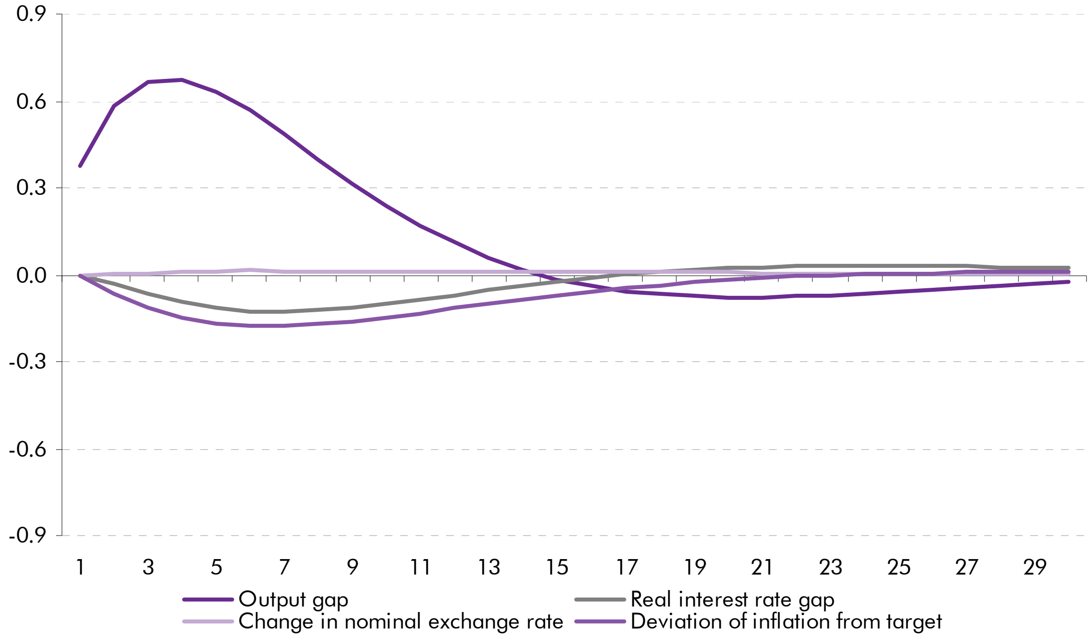


Chart D.2: Response of effective interest rate to shocks

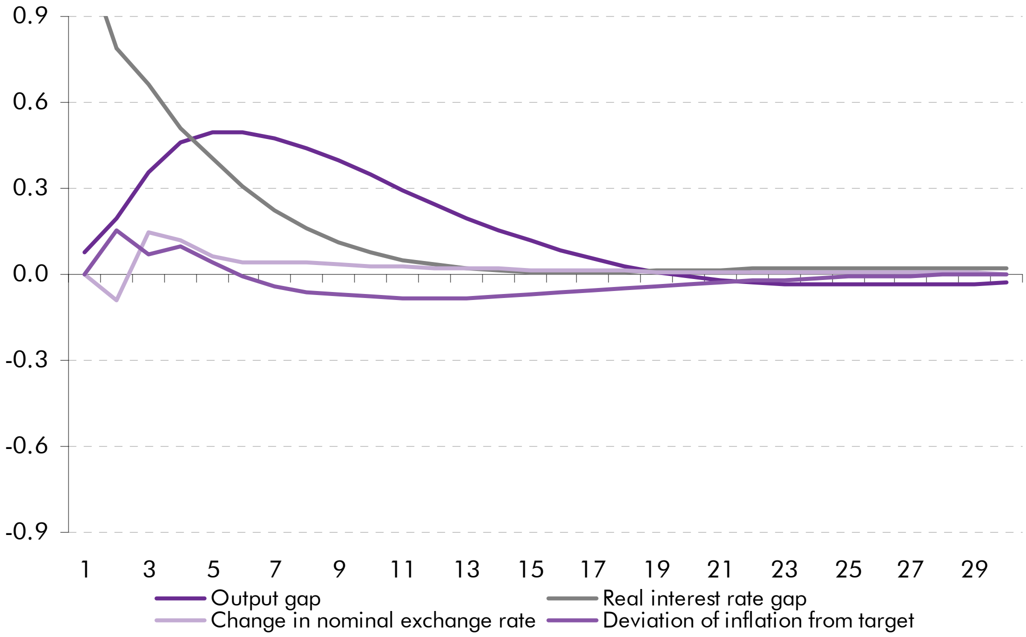


Chart D.3: Response of exchange rate gap to shocks

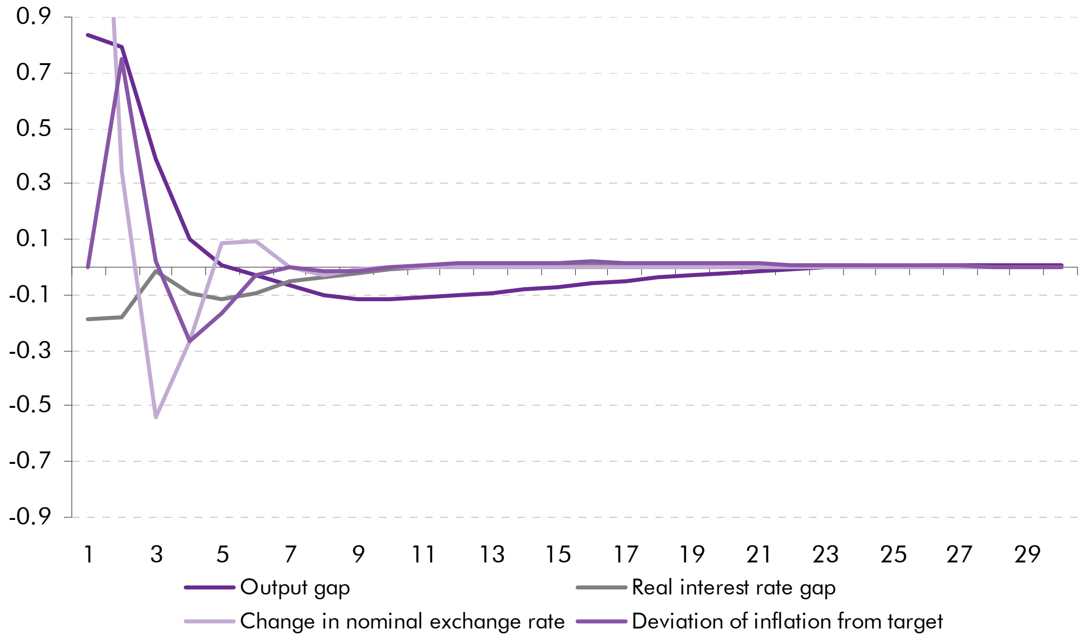
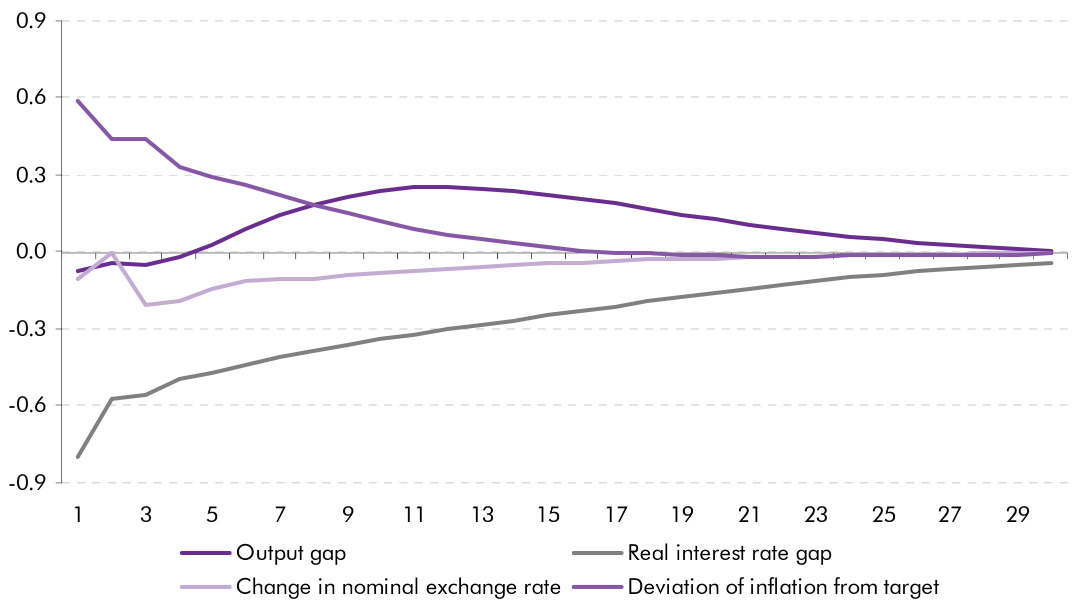


Chart D.4: Response of inflation gap to shocks



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