Cyclically adjusting the public finances

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Abstract

The specification of the Government’s fiscal mandate in cyclically-adjusted terms requires the OBR to make an assessment of the effect of the economic cycle on the public finances. These estimates are generally produced for the main fiscal aggregates using cyclical adjustment coefficients. In this paper we reassess the size of the cyclical adjustment coefficients both by revisiting previous Treasury analysis and by considering a range of other approaches. Our estimates suggest a contemporaneous cyclical adjustment coefficient to the output gap for net borrowing and current budget of 0.5 and a lagged coefficient of 0.2, which are the same as the coefficients the OBR has used in its forecasts to date. We use these coefficients and the OBR's historical output gap series to produce an updated historical series for structural net borrowing. We also analyse the effect that fluctuations in asset prices and property transactions, which are not related to the economic cycle, could have on the public finances using two different approaches.

We are grateful for comments from colleagues at the Office for Budget Responsibility, the OBR’s Advisory Panel and colleagues at HMRC and DWP.

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1 Introduction

1.1 The Office for Budget Responsibility (OBR) has been tasked with judging whether the Government has a greater than 50 per cent probability of achieving the medium-term fiscal targets that it has set itself. The Government’s fiscal mandate is currently defined as the requirement “to balance the cyclically-adjusted current budget (CACB) by the end of a rolling, five-year period”.

1.2 The specification of the fiscal mandate in cyclically-adjusted terms requires the OBR to make an assessment of the effects of the economic cycle on the public finances. This is generally done by adjusting a given fiscal aggregate, such as net borrowing or the current budget, for the amount of spare capacity in the economy (the output gap) using cyclical adjustment coefficients.

1.3 To date, the OBR has adopted the Treasury’s approach to cyclical adjustment as presented in the 2008 Treasury working paper: Public Finances and the cycle. In this paper we reassess the size of the cyclical adjustment coefficients. We do this by revisiting the previous Treasury analysis and by considering a range of alternative approaches to cyclical adjustment.

Estimating cyclically-adjusted fiscal aggregates

1.4 The cyclical position of the economy is likely to have an impact on public sector receipts and expenditure. If the economy is operating below its potential (i.e., there is a negative ‘output gap’) then, other things equal, there is likely to be higher expenditure on items such as jobseekers allowance. Similarly, there are likely to be lower receipts from sources such as income tax, corporation tax and VAT, due to lower labour income, corporate profits and consumer spending, respectively.

1.5 This implies that government borrowing will typically tend to be higher when output is below its potential level, and lower when output is above its potential. Adjusting for the impact of the cycle on the public finances provides an estimate of the ‘structural’ position of the public finances, once the temporary effects of the economic cycle have been removed.

1.6 The cyclical adjustment coefficients used to make this adjustment are derived by analysing the past relationship between the output gap and the fiscal position. As explained in the OBR’s Economic and fiscal outlooks (EFO), the estimates of these coefficients are highly uncertain for a number of reasons:
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- the output gap is not directly observable, so there is no historical ‘fact’ from which to estimate the coefficients;
- the number of observations on which to base coefficient estimates is limited;
- the fiscal position is affected by events that do not necessarily move in line with the cycle, such as one-off fiscal policy adjustments and movements in commodity and asset prices; and
- insofar as the current economic cycle differs from the average cycle, the relationship between the public finances and the output gap over the course of that cycle will not be captured in the coefficients.

1.7 In this paper we attempt to address these uncertainties as far as possible. We compare the results from several approaches to estimating the cyclical adjustment coefficients. In our central estimates we make use of the historical output gap series published in the OBR’s Working paper No. 1: Estimating the UK’s historical output gap, but we test the sensitivity of these results to alternative output gap series. We extend our analysis by using the structural VAR approach to consider the potential feedback from fiscal policy to the output gap. We also analyse the potential impact on the fiscal position from temporary factors that may not be correlated with the economic cycle, in particular asset prices and transactions.

Main conclusions

1.8 Our estimates suggest a contemporaneous cyclical adjustment coefficient of 0.5 and a lagged coefficient on the previous year of 0.2. These are the same coefficients that have been used in EFOs to date. We have used these coefficients and the OBR’s historical output gap series to produce an updated historical series for structural net borrowing. The change in the output gap series means that structural borrowing appears somewhat higher in the 1990s and in the run up to the 2008 financial crisis than previous Treasury estimates have implied.

Structure of the paper

1.9 The analysis in this paper is structured as follows:

- in Chapter 2 we estimate cyclical adjustment coefficients using the previous Treasury approach and the approach used by the Organisation for Economic Cooperation and Development (OECD). Both approaches are based on econometric estimation of the past relationship between the output gap and the fiscal position;
Chapter 3 considers some extensions to these approaches. We estimate a structural vector autoregressive model (SVAR) which attempts to address the endogeneity of the output gap and fiscal policy. We also look at the approach used by the European Central Bank (ECB) which attempts to correct for the impact of changes in the composition of GDP;

Chapter 4 analyses the potential temporary effects on the fiscal position of fluctuations in asset market prices and transactions, which may not be correlated with the economic cycle; and

Chapter 5 summarises and sets out our conclusions.
2 Estimating cyclical adjustment coefficients

2.1 This chapter explains and presents the results from two econometric approaches used to cyclically adjust the public finances. To date in the OBR’s forecasts of the public finances it has adopted the approach first set out in detail in a Treasury Occasional paper published in 1995, Public finances and the cycle. The results have since been updated by the Treasury on regular occasions – most recently in 2008 – but the underlying methodology has remained broadly unchanged.1 We call this the ‘one-step’ approach as it involves regressing public expenditure and receipts directly on the output gap.

2.2 We then consider a more widely-used methodology developed by the OECD. We label this the ‘two-step’ approach as it involves first regressing the tax/expenditure economic base (for example, total labour income as the base for income tax receipts) on the output gap, and then estimating the responsiveness of the tax/expenditure stream to its base. The OECD applies this approach across a number of countries and therefore makes a number of simplifying assumptions to do so. We are able to tailor the approach to the UK using country-specific models and data.

2.3 We use both approaches to estimate cyclical adjustment coefficients for the UK using the latest available datasets. We then assess the robustness and the relative merits of the two approaches. For our central estimates we have used the output gap series published in the OBR’s Working paper No. 1: Estimating the UK’s historical output gap, but we also consider the sensitivity of the results to alternative historical output gap series.

2.4 The approaches produce cyclical adjustment coefficients consistent with adjusting public sector net borrowing (PSNB) figures. However, since the difference between PSNB and the current budget balance is mainly public investment, which is small as a share of GDP, the coefficients can also be used to cyclically adjust the current budget.

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One-step approach

Overview of method

2.5 The underlying methodology of the one-step approach is relatively simple. Expenditure and revenue expressed as ratios to GDP over the past 30 years are regressed against estimates of contemporaneous and lagged output gaps. The coefficients in the equations indicate the average responsiveness of the public finances to the economic cycle.

2.6 In its analysis, HM Treasury adjusted the results of the regression analysis to allow for prior expectations based on structural changes in tax and expenditure systems that might not be picked up fully in the regression analysis. The adjusted coefficients were used by HM Treasury, and subsequently by the OBR, to adjust nominal fiscal aggregates (expressed as ratios to nominal GDP) for the impact of the economy’s position in the cycle.

2.7 There are a number of theoretical and practical difficulties with implementing this straightforward approach. For example, the use of historical data means that the estimates reflect the average effect of changes in the output gap on the public finances over previous cycles. If the current economic cycle differs from the average cycle, the relationship between the public finances and the output gap over the course of that cycle will not be captured in the coefficients. We therefore conduct some sensitivity analysis of the results to the choice of time period by looking at the results for individual cycles and recursive estimation of the equations. The results are discussed in Annex A.

2.8 Another problem with this approach is that ideally it is necessary to remove the effects of discretionary fiscal policy changes on the data series of government expenditure and receipts used in the regressions. Otherwise the regression may capture changes in receipts and expenditure driven by policy choices. In practice, as discussed below, adjusting data for policy effects is very difficult and is only really feasible for receipts, although some attempt is made to model policy effects in the expenditure regressions.2

2.9 The following sections discuss, in turn, the approach to modelling the aggregate and individual tax receipts effects, the expenditure effect and how to combine the

---

2 Note that this approach is similar to what has come to be termed the “narrative” approach to the identification of fiscal policy (see for example Romer & Romer (2010)). Indeed the dataset of discretionary policy decisions that we have made available on our website alongside this paper should prove to be a useful tool for further research in this area.
results into a single estimate of cyclical adjustment coefficients that can be applied to net borrowing and the current budget.

Receipts and the cycle

2.10 To estimate the effects of the economic cycle on tax receipts we begin by constructing a policy-adjusted dataset for the bulk of tax receipts using published costings of tax policy measures since 1970 (see Box 2.1). In theory, removing the estimated policy costings from the individual (and aggregate) receipts series leaves us with a tax series that should be predominantly driven by movements in the economic cycle. Chart 2.1 shows the results of this exercise.3

Chart 2.1: Indexed tax to GDP ratios

The adjusted series follows our estimate of the economic cycle a little more closely, though the correlation is not very strong. The difference between the two series also appears to be largely related to the cyclical position of the economy:

- from 1984-85 to 1989-90: a gap opens up as the unadjusted series falls, while the adjusted series increases. This pattern is consistent with a cyclical increase in tax receipts that is more than offset by procyclical discretionary policy loosening;

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3 The chart starts in 1973 due to the unavailability of more detailed information on tax receipts prior to this year, which restricts the effective size of our sample period.
• from 1990-91 to 1993-94: both series fall at approximately the same rate, due to a cyclical fall in tax receipts; and

• from 1994-95 to 2000-01: the gap closes as (milder) procyclical discretionary policy tightening boosts the unadjusted series.

2.12 Note that this series only encompasses an average of around 85 per cent of total tax receipts and covers the major tax heads: income tax & NICs, VAT, corporation tax, fuel duty (plus VED), capital taxes and excise duties. This is because Budget costings for the other principal elements (e.g. local authority taxes) are not available on a consistent basis. We therefore exclude local authority taxes and other receipts (mainly interest dividends, trading surpluses and rent) from this analysis. However, this is a slightly larger wider tax aggregate than used in previous Treasury analyses, which also excluded capital taxes. Box 2.1 provides more detail of the method we use to produce the policy-adjusted series and the uncertainties involved.

2.13 Once the dataset has been policy-adjusted it is also necessary to adjust for large, ‘one-off’, fiscal operations that can affect the calculation of the cyclically-adjusted fiscal balances even though they have no, or very little, implications for the fiscal stance. The only example in the UK that requires a specific adjustment is the auctioning of third generation mobile telephone licenses. The new classification treatment of these receipts as a net capital transfer will reduce PSNB (but not tax receipts) by a one-off amount of £22.5bn in 2000-01 – an effect which should be removed from the calculation of cyclically-adjusted borrowing.5

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4 The proportion of total tax receipts accounted for by these tax heads rises through the 1980s from 75 per cent to over 90 per cent at the start of the 1990s.

Box 2.1: Constructing the policy-adjusted tax receipts dataset

To compile our policy-adjusted tax receipts series we have constructed a database of all of the larger tax policy measures of the last 42 years. These policy measures are listed in the tables which set out the costs of new measures published by the Treasury at Budgets and other fiscal events since Budget 1970. This approach therefore assumes that these initial policy costings were accurate, although there are significant uncertainties around many policy costings, and their accuracy is often very difficult to assess ex-post.

In the interests of efficiency, we have typically only included policy measures that were costed in excess of plus or minus £50 million in at least one year. In some cases, smaller measures were grouped together where it was easy to do so and in later years smaller measures for the smaller tax heads have also been included. The Budget tables typically only show costings for the next two years, although more recent Budgets have shown up to five years. So these costings were then extrapolated for the whole data period using the growth rate of nominal GDP. This is another weakness of this approach but there is no obvious better alternative.

The policy-adjusted receipts series is then constructed simply by subtracting these costings from the relevant data series of actual receipts. With the very significant caveat of the uncertainties set out above, the policy-adjusted series is therefore theoretically showing the level of receipts that would have been recorded if no tax policy changes had been made after 1970.

Despite the inherent difficulties with this approach, the headline results are reasonably intuitive. Taking the period as a whole, the fact that the two lines meet at the end as well as the beginning suggests that discretionary tax increases and decreases have broadly offset each other over the period. Discretionary income tax measures have reduced tax receipts to counter the effects of fiscal drag, while indirect tax measures, especially for VAT and fuel duties, have boosted receipts. In the case of VAT, this reflects a general shift in the burden of taxation from direct to indirect taxes, while the fuel duty measures have partially offset the effects of a declining tax base as fuel efficiency has improved.

The increase in the tax-to-GDP ratio across the sample period largely reflects the choice of starting point in 1973-74, as the ratio was at its cyclical low at this point, following a year of very rapid growth in GDP.

* The database is available on the OBR’s website at www.budgetresponsibility.independent.gov.uk.
Estimating cyclical adjustment coefficients

2.14 The policy adjusted series is then included in the following simple regression:

\[(TR / Y)_t = \alpha + \beta_1 y_{gap,t} + \beta_2 y_{gap,t-1} + \gamma^*_t + \epsilon_t, \quad (1)\]

2.15 Where \((TR / Y)_t\) is policy adjusted tax receipts as a share of nominal GDP, \(Y_t\), \(\alpha\) is a constant, \(y_{gap}\) is the output gap, \(Y^*_t\) is the level of potential output (in logs) and \(\epsilon_t\) is an error term. The regression is estimated using ordinary least squares (OLS) over the sample period, financial years, 1972-2010. The coefficients on the output gap and the lagged output gap are the coefficients of interest and indicate the responsiveness of tax receipts to the cycle. The lagged output gap is included in the specification on the basis of prior expectations that there are likely to be lags from changes in the output gap to changes in receipts. This may be due to lags in the economy - for example changes in the labour market may lag changes in output. There are also lags in the tax system. For example self-assessment tax is paid some period after earnings are received, and these receipts are not accrued back to the earlier period as happens for income tax receipts. The potential output variable can be thought of as a proxy for the effects of real fiscal drag.6

2.16 The use of a policy-adjusted series tackles one possible source of endogeneity, but another remains: there might also be a simultaneous relationship between expenditure, receipts and the output gap. The output gap can be affected by fiscal policy working through the level of receipts and expenditure. The level of government expenditure (the consumption element of which is scored in GDP) is also likely to depend in part on the level of tax receipts, and vice versa.

2.17 One possible method for addressing this issue is to estimate the equation using instrumental variables. But in practice it can often be difficult to identify strong instruments. This was the approach used in the Treasury’s 2008 working paper, using the world output gap and world interest rates as instruments. This detected no significant evidence of bias, in line with the conclusions of Darby and Melitz (2008). An alternative method, discussed in Chapter 3, is to adopt a more formal structural modelling approach and estimate a structural vector autoregressive model (SVAR).

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6 Fiscal drag is the name given to the tendency for tax receipts as a percentage of GDP to increase over time. This is due to the progressive nature of the tax system, whereby the average tax rate increases the more income is earned. Nominal fiscal drag occurs when inflation pushes incomes up; real fiscal drag occurs when wages rise faster than inflation due to productivity growth.
Estimating cyclical adjustment coefficients

Results for tax receipts and the cycle

Aggregate taxes

2.18 The results of estimating the aggregate tax receipts equation are shown in Table 2.1, and presented alongside those of the previous Treasury analysis to facilitate comparison. The updated estimates might be expected to differ from the Treasury results due to the use of a longer sample period, which now includes movements in the output gap caused by the recent recession, and the use of the OBR estimate of the output gap, as opposed to the Treasury’s construction of this variable.\(^7\)

Table 2.1: Aggregate tax receipts (per cent of GDP)

<table>
<thead>
<tr>
<th>Year</th>
<th>Constant</th>
<th>Output gap</th>
<th>Output gap (-1)</th>
<th>Trend GDP</th>
<th>R squared</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008</td>
<td>19.9</td>
<td>-</td>
<td>0.18</td>
<td>0.08</td>
<td>0.91</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td>(0.6)</td>
<td></td>
<td>(2.9)</td>
<td>(-0.0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OBR 2012</td>
<td>-27.9</td>
<td>-</td>
<td>0.14</td>
<td>4.5</td>
<td>0.43</td>
<td>1.71</td>
</tr>
<tr>
<td></td>
<td>(-1.9)</td>
<td></td>
<td>(1.0)</td>
<td>(4.5)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

T-statistics in brackets

2.19 However, while the results differ in a few respects from the previous Treasury estimate, the coefficients on the key variable – the output gap – are of a similar magnitude. The results continue to indicate no significant relationship with the current output gap with a slightly smaller coefficient on the lagged output gap. The divergent estimates of the coefficient on potential output appear to be due to an anomalous result in the Treasury’s 2008 analysis, as the previous analyses published in 1999 and 2003 both reported coefficients closer in magnitude to our updated estimates. The size of the coefficient implies that real fiscal drag increases the tax to GDP ratio by around 0.1 per cent a year.

2.20 The fall in the R-squared statistic – a measure of the explanatory power of the regression – is likely to be due to the extension of the end of the sample period from 2006 to 2010. That the equation does not fit very well over the recent recession suggests that the responsiveness indicated by the regression coefficients may not be a good guide to the current responsiveness of tax receipts to the economic cycle.

2.21 In its 2008 paper the Treasury judged that, despite the results of the regression analysis, it is reasonable to allow for some contemporaneous effect between the cycle and tax receipts. For example, the Treasury suggested that the introduction

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of quarterly instalment payments (QIPs) of corporation tax for large companies from 1999 might have led to more timely responsiveness of receipts to the cycle. The Treasury therefore adjusted the regression results and placed a coefficient of 0.1 on the contemporaneous output gap, and a coefficient of 0.1 on the lagged output gap. This is broadly equivalent to bringing forward half the impact associated with the estimated regression coefficient on the lagged output gap.

2.22 This is a conclusion shared by other analyses of cyclical adjustment. For example, the OECD’s implementation of the two-step approach (Girouard and André (2005)) comments that exact lag structures for UK corporate and personal income tax are not known, and they may vary significantly over time. On the basis of judgement, however, the OECD also assumes a two year adjustment period, with equal weight on the contemporaneous and lagged effect.

Individual taxes

2.23 To complement the top-down regression analysis, and to gain further insight into the cyclicity of tax receipts, we carry out similar regression analysis for each of the individual components of the aggregate tax receipts series. The responsiveness to the output gap could differ across the tax base, and individual component regressions (as are also used in the two-step approach), can provide useful additional information on the relationship between the cycle and aggregate receipts. The results are shown in Tables 2.2 to 2.8.

Table 2.2: Income tax and NICs receipts (per cent of GDP)

<table>
<thead>
<tr>
<th></th>
<th>Constant</th>
<th>Output gap</th>
<th>Output gap</th>
<th>Trend GDP</th>
<th>R squared</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>(-1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2008</td>
<td>20.0</td>
<td>-0.14</td>
<td>0.18</td>
<td>6.0</td>
<td>0.93</td>
<td>0.62</td>
</tr>
<tr>
<td>OBR 2012</td>
<td>98.8</td>
<td>-0.25</td>
<td>-</td>
<td>8.6</td>
<td>0.79</td>
<td>1.34</td>
</tr>
<tr>
<td>(-92)</td>
<td>(-2.4)</td>
<td></td>
<td></td>
<td>(11.0)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.3: Non-oil corporation tax receipts (per cent of GDP)

<table>
<thead>
<tr>
<th></th>
<th>Constant</th>
<th>Output gap</th>
<th>Output gap</th>
<th>Trend GDP</th>
<th>R squared</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>(-1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2008</td>
<td>1.5</td>
<td>0.08</td>
<td>0.10</td>
<td>-1.1</td>
<td>0.78</td>
<td>0.34</td>
</tr>
<tr>
<td>OBR 2012</td>
<td>2.9</td>
<td>0.17</td>
<td>-</td>
<td>0.27</td>
<td>0.58</td>
<td></td>
</tr>
<tr>
<td>(31.0)</td>
<td>(3.7)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Estimating cyclical adjustment coefficients

#### Table 2.4: VAT receipts (per cent of GDP)

<table>
<thead>
<tr>
<th></th>
<th>Constant</th>
<th>Output gap</th>
<th>Output gap (-1)</th>
<th>Trend GDP</th>
<th>R squared</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008</td>
<td>4.6</td>
<td>0.05</td>
<td>-</td>
<td>-</td>
<td>0.83</td>
<td>0.34</td>
</tr>
<tr>
<td>OBR 2012</td>
<td>2.9</td>
<td>-0.04</td>
<td>-</td>
<td>-</td>
<td>0.02</td>
<td>0.52</td>
</tr>
</tbody>
</table>

#### Table 2.5: Excise motor tax receipts (per cent of GDP)

<table>
<thead>
<tr>
<th></th>
<th>Constant</th>
<th>Output gap</th>
<th>Output gap (-1)</th>
<th>Trend GDP</th>
<th>R squared</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008</td>
<td>31.1</td>
<td>-0.04</td>
<td>-</td>
<td>-2.2</td>
<td>0.91</td>
<td>0.16</td>
</tr>
<tr>
<td>OBR 2012</td>
<td>26.4</td>
<td>0.08</td>
<td>-</td>
<td>-1.9</td>
<td>0.80</td>
<td>0.29</td>
</tr>
</tbody>
</table>

#### Table 2.6: Other excise tax receipts (per cent of GDP)

<table>
<thead>
<tr>
<th></th>
<th>Constant</th>
<th>Output gap</th>
<th>Output gap (-1)</th>
<th>Trend GDP</th>
<th>R squared</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008</td>
<td>65.2</td>
<td>-</td>
<td>-</td>
<td>-4.7</td>
<td>0.97</td>
<td>0.37</td>
</tr>
<tr>
<td>OBR 2012</td>
<td>48.0</td>
<td>0.08</td>
<td>-</td>
<td>-3.5</td>
<td>0.89</td>
<td>0.36</td>
</tr>
</tbody>
</table>

#### Table 2.7: Capital taxes (per cent of GDP)

<table>
<thead>
<tr>
<th></th>
<th>Constant</th>
<th>Output gap</th>
<th>Output gap (-1)</th>
<th>Trend GDP</th>
<th>R squared</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>OBR 2012</td>
<td>-15.5</td>
<td>0.05</td>
<td>-</td>
<td>1.2</td>
<td>0.77</td>
<td>0.20</td>
</tr>
</tbody>
</table>

#### Table 2.8: Sum of output gap coefficients (per cent of GDP)

<table>
<thead>
<tr>
<th></th>
<th>Output gap</th>
<th>Output gap (-1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008</td>
<td>-0.05</td>
<td>0.28</td>
</tr>
<tr>
<td>OBR 2012</td>
<td>-0.04</td>
<td>0.16</td>
</tr>
</tbody>
</table>

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8 Fuel duty and vehicle excise duty.
9 Tobacco, alcohol and betting duties.
10 Where statistically significant i.e. excluding the VAT coefficient in the OBR equation.
Our results are again broadly similar to those of the 2008 Treasury paper. A negative sign on the contemporaneous output gap in the income tax equation is not immediately intuitive, but probably reflects cyclicality in the labour share of income, although there is no longer a partial counterbalance by the finding of a statistically significant positive relationship for the lagged output gap. The coefficient on trend GDP is consistent with fiscal drag of around 0.2 per cent a year.

The corporate tax equation now shows a bigger contemporaneous effect than previously, which lends some support to the adjustment made by the Treasury for the introduction of QIPs, discussed in paragraph 2.21. The excise equations point to a larger effect from the lagged output gap, which offset the loss of significance in the income tax equation. The negative coefficients on trend GDP reflect the downward trend in the share of GDP of these taxes. The VAT equation performs extremely poorly, with a complete absence of statistical significance and explanatory power. Overall, the result is that the sum of the individual estimates shown in Table 2.8 is now smaller than in the 2008 analysis, but consistent with the results of the aggregate equation.

On the basis of these updated regression results, there seems to be little evidence pointing strongly to a different conclusion from the Treasury’s 2008 analysis of the cyclicality of receipts. However, the instability of the individual equation estimates raises questions about the robustness of the one-step approach.

Expenditure and the cycle

Frequent changes in the coverage and structure of expenditure programmes mean that it is not possible to adjust the public expenditure series for policy effects in the same way as we did for receipts. For example, aside from rate changes, the basic structures of income tax and VAT have remained largely unchanged, whereas it is not possible to track a consistent series through time for programmes such as tax credits, which are responsible for a large share of expenditure, but are a relative recent innovation in their current form. It is therefore necessary to try to capture some of the possible expenditure policy effects in the modelling approach. The Treasury’s choice of expenditure regression therefore has a slightly different specification to the receipts regression:

\[
\frac{TME}{Y}_t = \alpha + \beta_1 \frac{TME}{Y}_{t-1} + \beta_2 \frac{TME}{Y}_{t-2} + \gamma \text{output gap}_t + \delta_1 T75 + \delta_2 T02 + \epsilon_t \quad (2)
\]

Where \(\frac{TME}{Y}_t\) is Total Managed Expenditure (TME) as a share of nominal GDP, \(Y_t\), \(\alpha\) is a constant, \(\text{output gap}_t\) is the output gap, T75 and T02 are time trends starting in 1975-76 and 2002-03 respectively, and \(\epsilon_t\) is an error term. The
regression is estimated using ordinary least squares (OLS) over the sample period, financial years, 1972-2010.

2.29 The inclusion of lagged dependent variables is designed to capture endogenous policy responses. For example, policymakers may decide to increase social security expenditure, over and above the ‘automatic’ increase in unemployment related benefits, once it becomes clear the economy is in a downturn. But note that under these assumptions (i.e. that the policy response is countercyclical and responds with a lag) the long-run response of spending to the economic cycle implied by the regression is likely to be an overestimate.¹¹

2.30 The inclusion of a number of time trends again follows the previous Treasury approach and is designed to capture potentially more long-lasting ‘structural’ policy effects. For example, the inclusion of a trend from 2002 is intended to account for a discretionary decision to increase the level of expenditure over subsequent years, announced in Budget 2002.¹²

2.31 The choice of dependent variable in the expenditure regression is total spending, also known as Total Managed Expenditure (TME), which the Treasury split out into Annually Managed Expenditure (AME) and Departmental Expenditure Limits (DEL). DEL is mainly composed of expenditure on the provision of public services such as education and health, which has typically been fixed in cash terms in multi-year plans and is therefore not directly linked to the economic cycle. AME will typically follow the economic cycle more closely as it contains elements such as social security expenditure which are directly linked to the level of unemployment and earnings. However, these elements are generally quite small relative to the rest of public expenditure.

2.32 Therefore, with much of expenditure broadly fixed in nominal terms we might expect that, when measured as a ratio to GDP, expenditure will be sensitive to the cycle principally through a ‘denominator effect’. In other words, movements in GDP will be the main driver of the contemporaneous effect of the cycle on the public finances. For example, a fall in GDP will automatically increase the share of TME to GDP if the level of TME is broadly stable in nominal terms.

2.33 The ratio of TME to GDP has averaged 43 per cent across the sample period, which implies that a one per cent increase in output relative to trend would reduce the share of TME as a percentage of GDP by around 0.4 percentage

¹¹ Calculated as \( \gamma_1 / (1 - \beta_1 - \beta_2) \).

¹² In particular, Budget 2002 announced significant increases in health spending to an average annual growth rate of 7.5 per cent over the five years to 2007/08.
points in the first year. The regression results reported in Table 2.9 provide empirical support for this conclusion.

Table 2.9: Total managed expenditure (per cent of GDP)

<table>
<thead>
<tr>
<th></th>
<th>Constant</th>
<th>TME(-1)</th>
<th>TME(-2)</th>
<th>Output gap</th>
<th>T75</th>
<th>T02</th>
<th>R-squared</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008</td>
<td>30.1</td>
<td>0.79</td>
<td>-0.41</td>
<td>-0.34</td>
<td>-0.22</td>
<td>1.02</td>
<td>0.97</td>
<td>0.76</td>
</tr>
<tr>
<td>OBR 2012</td>
<td>29.1</td>
<td>0.70</td>
<td>-0.31</td>
<td>-0.44</td>
<td>-0.24</td>
<td>0.88</td>
<td>0.91</td>
<td>1.32</td>
</tr>
</tbody>
</table>

The relationship between the output gap and TME appears to be relatively stable as the estimated coefficients are similar to previous Treasury analysis. The increase in the coefficient on the output gap is likely to be due to the extension of the sample period to cover the recent recession. This would be the case if the increase in spending in response to recent cyclical weakness has been larger than the historical average, perhaps due to countercyclical discretionary policy decisions for which we are unable to adjust. The results imply that a one per cent rise in GDP, relative to potential output, reduces the TME to GDP ratio by 0.44 percentage points in the first year. This is in-line with the prior expectation of a coefficient of around 0.4 due to the denominator effect.

However, the equation also includes dynamic effects and the estimated lagged and long-run responses are shown in Table 2.10. The expenditure response rises to 0.7 per cent in the second year and remains around that level over the longer term. This is above the 0.5 adjustment for expenditure in the Treasury’s 2008 paper, although as discussed earlier, the coefficient may be subject to some upward bias if the endogenous policy response captured by the lagged terms is countercyclical (i.e. spending is increased when GDP falls).

Table 2.10: Dynamic response of the TME to GDP ratio to a 1 per cent increase in the output gap

<table>
<thead>
<tr>
<th></th>
<th>T</th>
<th>T+1</th>
<th>Long run</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008</td>
<td>-0.34</td>
<td>-0.61</td>
<td>-0.55</td>
</tr>
<tr>
<td>OBR 2012</td>
<td>-0.44</td>
<td>-0.74</td>
<td>-0.73</td>
</tr>
</tbody>
</table>

Cyclical social security

As discussed above some elements of AME expenditure – in particular Jobseeker’s Allowance and other income related elements of the social security system – are likely to be closely linked to movements in the cycle. A separate regression is therefore estimated to test for the effect of ‘cyclical social security’ expenditure, which we define as all DWP income-related benefits and tax credits.
administered by HMRC. Costings for some policy measures related to these benefits are available, but we have not attempted to adjust the series for the subset of measures that are available as the output would only be a partially-adjusted series. The results are shown in Table 2.11.

Table 2.11: Cyclical social security (per cent of GDP)

<table>
<thead>
<tr>
<th>Year</th>
<th>Constant</th>
<th>Output gap (-1)</th>
<th>Time</th>
<th>R squared</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008</td>
<td>1.18</td>
<td>-0.07</td>
<td>0.28</td>
<td>0.34</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4.8)</td>
<td>(-3.0)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OBR 2012</td>
<td>2.20</td>
<td>-0.18</td>
<td>0.07</td>
<td>0.79</td>
<td>0.46</td>
</tr>
<tr>
<td></td>
<td>(14.3)</td>
<td>(-4.9)</td>
<td>(9.7)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The results indicate a stronger relationship than in the previous Treasury analysis, or a coefficient of 0.18, which is likely to be due to the wider definition of cyclical social security used in our analysis. If we use the narrower measure of unemployment related benefits as the dependent variable, then we find that the coefficient on the output gap drops back to 0.1. The slightly larger coefficient is also consistent with the rise in the cyclical response of total expenditure in the t+1 period, given by the dynamic terms in the TME regression. The inclusion of a time trend, which accounts for the steady increase in this measure of the level of social security expenditure through the sample period, improves the fit of the regression.

Unlike the individual tax receipts exercise, simply adding the coefficients from the TME and cyclical social security regressions would likely lead to some double-counting, as this social security expenditure is already included in the TME measure. This was confirmed by repeating the TME regression in Table 2.9, but substituting a TME excluding cyclical social security measure as the dependent variable. In this case the coefficient on the output gap drops from 0.43 to 0.40, as shown in Table 2.12.

Table 2.12: TME excluding cyclical social security (per cent of GDP)

<table>
<thead>
<tr>
<th>Year</th>
<th>Constant</th>
<th>TMEx (-1)</th>
<th>TMEx (-2)</th>
<th>Output gap</th>
<th>T75</th>
<th>T02</th>
<th>R-squared</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>OBR 2012</td>
<td>31.6</td>
<td>0.62</td>
<td>-0.30</td>
<td>-0.40</td>
<td>-0.32</td>
<td>1.05</td>
<td>0.93</td>
<td>1.23</td>
</tr>
<tr>
<td></td>
<td>(5.7)</td>
<td>(3.2)</td>
<td>(-2.1)</td>
<td>(-2.8)</td>
<td>(-5.3)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2.39 The Treasury analysis also investigated the possibility of debt interest payments being cyclical. However, the coefficient on the output gap in the debt interest regression was not statistically significant in either the Treasury’s 2003 or 2008 analysis. We found the same result so we have chosen not to report the detail here. This may be because if economic growth is above trend then interest rates will tend to rise (increasing debt interest payments) but borrowing will tend to fall (reducing debt interest payments). A large proportion of debt interest costs will also reflect payments on the historical stock of debt. Any cyclical influence would only affect the debt that is being refinanced at the time. Therefore, cyclical effects on debt interest payments are likely to be small but persistent.

Updated estimates of the cyclical adjustment coefficients

2.40 The main results of all the regression equations are summarised in Tables 2.13 to 2.15. Table 2.13 shows the unadjusted results of the econometric estimates in the Treasury’s 2008 analysis, and the adjustments made by the Treasury to produce the cyclical adjustment coefficients is given in Table 2.14. Finally Table 2.15 shows the results of our updated analysis.

2.41 The results of our regressions are broadly similar to those in the Treasury’s 2008 paper. Our estimates produce slightly higher coefficients on both the contemporaneous output gap and the lagged output gap. However, if the same sets of adjustments are made to our results as were made in the Treasury paper then a very similar set of cyclical adjustment coefficients would be produced. As we have discussed above, the arguments that the Treasury cited to justify these adjustments – on the expected size of the expenditure ‘denominator’ effect and the expected contemporaneous effect of the output gap on tax – continue to have some support in the empirical results.

2.42 However there are a number of limitations with this approach. Policy adjustment is only feasible on the revenue side and relies on a number of strong assumptions (see Box 2.1). The results of the regression analysis are not very robust in a number of cases, and our sensitivity tests in Annex A indicate that the results are also not very robust to the choice of sample period. In particular, this means that we may only be capturing (imprecisely) the average effect of the cycle on the public finances when we are really interested in the marginal effect at the current juncture. The OECD or two-step approach discussed in the next section seems better suited to capture this effect.
Estimating cyclical adjustment coefficients

Table 2.13: Previous HM Treasury 2008 econometric results

<table>
<thead>
<tr>
<th>Fiscal aggregate</th>
<th>Output gap</th>
<th>Lagged output gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>CA expenditure</td>
<td>TME</td>
<td>+ 0.34 + 0.07</td>
</tr>
<tr>
<td>CA receipts</td>
<td>PSCR</td>
<td>-</td>
</tr>
<tr>
<td>CAPSNB</td>
<td>PSNB</td>
<td>+ 0.34 + 0.18</td>
</tr>
</tbody>
</table>

Table 2.14: HM Treasury 2008 cyclical-adjustment coefficients after adjustment

<table>
<thead>
<tr>
<th>Fiscal aggregate</th>
<th>Output gap</th>
<th>Lagged output gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>CA expenditure</td>
<td>TME</td>
<td>+ 0.4 + 0.1</td>
</tr>
<tr>
<td>CA receipts</td>
<td>PSCR</td>
<td>- 0.1 - 0.1</td>
</tr>
<tr>
<td>CAPSNB</td>
<td>PSNB</td>
<td>+ 0.5 + 0.2</td>
</tr>
</tbody>
</table>

Table 2.15: Updated OBR econometric results and adjustments

<table>
<thead>
<tr>
<th>Fiscal aggregate</th>
<th>Output gap</th>
<th>Lagged output gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>CA expenditure</td>
<td>TME</td>
<td>+ 0.40 + 0.18</td>
</tr>
<tr>
<td>CA receipts</td>
<td>PSCR</td>
<td>-</td>
</tr>
<tr>
<td>CAPSNB</td>
<td>PSNB</td>
<td>+ 0.40 + 0.32</td>
</tr>
</tbody>
</table>

| Adjusted CAPSNB* | PSNB       | + 0.5 + 0.2       |

* Same adjustments as in Table 2.14 are applied

Two-step approach

This section looks at the approach developed and used by the OECD to cyclically adjust the fiscal aggregates of its member countries. The method estimates cyclical adjustment parameters for individual revenue and expenditure categories in two steps. The first step is to estimate how the economic ‘base’ (e.g. consumption and profits) of the tax/expenditure item responds to the output gap and the second step is to estimate how tax receipts and expenditure move with the relevant base. Those estimates are then combined to produce a single elasticity which tells us how much tax receipts and expenditure move with the output gap. These tax and expenditure estimates are aggregated to produce a

---

single set of cyclical adjustment coefficient for the fiscal aggregates, which are comparable to the coefficients derived in the one-step approach – see Figure 2.1.

Figure 2.1: Flow diagram of the OECD method

- Elasticity of tax/spending base to the output gap
- Aggregated elasticity of tax receipts and spending to the output gap
- The ratio of tax and spending to GDP
- Semi-elasticity of net borrowing

2.44 We make use of HMRC and DWP forecast models to estimate the elasticities of tax and expenditure to their relevant bases. They represent the marginal change in tax and expenditure consistent with current fiscal policy and are therefore invariant to the effects of historical discretionary policy measures. This is a major benefit compared to the one-step approach, given the difficulties of adjusting the revenue and expenditure series for the effects of policy.

2.45 However, these elasticities may themselves be cyclical. For example, effective tax rates may differ over the economic cycle due to movements in tax evasion, the composition of consumer spending, the use of losses in corporation tax and the greater use of part-time workers. These issues are discussed in Box 2.2.

2.46 This approach can also only be used where we can clearly identify both a base and the sensitivity of the particular tax or expenditure category to it. For example, this is the reason the OECD only takes account of unemployment related expenditure. The method could therefore underestimate cyclicality if other areas of social security expenditure that are not included in the analysis are also related to the cycle. In similar fashion to the one-step approach, the two-step approach is also vulnerable to the possibility that the endogeneity of fiscal policy may be biasing the results. This issue is addressed in Chapter 3.
Box 2.2: Effective tax rates and the cycle

A potential disadvantage with the two-step approach is that, unlike the one-step approach, it does not take into account other cyclical factors that might affect receipts but not the tax base. Possible examples would be cyclical movements in tax evasion (or speed of compliance), consumption patterns and the use of losses in corporation tax which could amplify the responsiveness of taxes to the cycle.

A recent IMF study suggests that tax revenue efficiency does indeed move with the business cycle. In particular a worsening of VAT efficiency is found to be driven by shifts in consumption patterns, a decrease in the share of standard rate consumption, and changes in tax evasion during contractions. The study also finds a correlation between personal income tax and social security tax efficiency and the output gap.

There is some evidence to suggest that other cyclical factors have affected receipts in the UK in the recent downturn. In particular:

- the VAT gap, the difference between the theoretical total VAT liability and actual cash receipts, increased in 2008-09 as the economy moved into recession. The VAT gap can indicate the degree of tax compliance. The VAT gap increased primarily because of a rise in unauthorised VAT debt and the use of the government’s time-to-pay scheme to spread tax payments over a longer time period. However, the VAT gap fell back to pre-recession levels in 2009-10 and 2010-11. There was also a fall in the share of standard rated consumption in 2008-09 and 2009-10;

- the effective tax rate on corporate profits was lowered by firms being able to carry back losses against recently paid tax related to previous years’ liabilities and carrying forward losses to be used when the firm returns to profitability. The carrying back of losses boosted repayments in 2009-10, while trading losses carried forward are expected to be remain higher than prior to the downturn for a prolonged period, particularly in the financial sector; and

- the effective tax rate on labour income was affected by the shift towards part-time work. Part-time workers generally face a lower marginal tax rate. In addition, high-paying sectors such as the financial sector tend to be more cyclical than other sectors, reducing the tax take from workers with high marginal tax rates.

\(^a\) Sancak, C. et al. (2010).

\(^b\) Tax efficiency: the share of tax revenues in the tax base, normalised by the standard tax rate.
Overview of method

2.47 Cyclically-adjusted net borrowing $b^*$, as a share of potential output\textsuperscript{16}, can be defined as cyclically adjusted government expenditure ($G^*$) minus cyclically adjusted receipts ($T^*$) minus: \[ b^* = \frac{G^* - \left( \sum_{i=1}^{n} T_{i}^* \right)}{Y^*} \] (3)

2.48 The cyclically-adjusted receipts and expenditure terms in equation (3) can be estimated using the elasticities of tax receipts ($\varepsilon_{i,\text{ygap}}$) and expenditure ($\varepsilon_{g,\text{ygap}}$) with respect to the output gap:

\[ T_{i}^* = \left( \frac{Y^*}{Y} \right)^{\varepsilon_{i,\text{ygap}}} \cdot T_{i} \] (4)

\[ G^* = \left( \frac{Y^*}{Y} \right)^{\varepsilon_{g,\text{ygap}}} \cdot G \] (5)

2.49 The elasticities are calculated using a two-step approach. On the revenue side the first step is to calculate the elasticity of revenue with respect to the relevant tax base ($t_{bi}$) and then to calculate the elasticity of the tax base to the output gap. The product of those elasticities gives the output elasticity of each tax category ($i$):

\[ \varepsilon_{t_{i},\text{ygap}} = \varepsilon_{t_{i},t_{bi}} \cdot \varepsilon_{t_{bi},\text{ygap}} \] (6)

2.50 Similarly on the expenditure side the elasticity of expenditure can be split into two components: the elasticity of expenditure with respect to its base (unemployment) and that of the base with respect to the output gap:

\[ \varepsilon_{g,\text{ygap}} = \varepsilon_{g,U} \cdot \varepsilon_{U,\text{ygap}} \] (7)

\textsuperscript{16} Note that this definition uses potential output as the denominator for the cyclically-adjusted budget balance; whereas the one-step approach uses actual output as the denominator. To compare the results of the two approaches therefore requires a little algebraic manipulation. The relationship between the two methods is discussed in more detail in Annex B.

\textsuperscript{17} Here $G^*$ is cyclically-adjusted government expenditure (the cyclical element is assumed to be dependent on unemployment); $T_{i}^*$ is cyclical adjusted tax category $i$; and $Y^*$ is the level of potential output.
The relevant tax and expenditure categories

2.51 The starting point for this approach is to identify which tax and expenditure categories are likely to have a cyclical element. The OECD identifies corporate tax, personal income tax, indirect tax and social security contributions on the receipts side, and unemployment benefits on the spending side. The economic bases that correspond to these tax categories are wages and salaries, corporate profits and consumer expenditure and the main economic base for unemployment benefits is the level of unemployment.

2.52 To provide a comparable and consistent estimate of cyclically-adjusted fiscal aggregates the OECD uses the same tax and spending categories and bases across all the countries that it covers. We are able to develop this approach by allowing for the particular structure of the UK tax and expenditure system.

2.53 We therefore identify income tax, national insurance contributions, non-oil and non-financial corporation tax, financial sector corporation tax, business rates, VAT, fuel duty, excise duties and capital taxes as potentially cyclical elements of receipts in the UK. On the expenditure side we identify Jobseeker’s Allowance and other DWP income-related benefits paid to jobseekers (such as housing and council tax benefits) as directly related to unemployment and therefore cyclical. The relevant economic bases for these items are set out in Table 2.16.

Table 2.16: Tax and expenditure categories and bases

<table>
<thead>
<tr>
<th>Tax /expenditure category</th>
<th>Tax /expenditure base</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income tax</td>
<td>Wages and salaries</td>
</tr>
<tr>
<td>National insurance contributions</td>
<td>Wages and salaries</td>
</tr>
<tr>
<td>Non-oil, non-financial corporation tax</td>
<td>Non-oil, non-financial corporate profits</td>
</tr>
<tr>
<td>Financial corporation tax</td>
<td>Financial corporate profits</td>
</tr>
<tr>
<td>Business rates</td>
<td>Output</td>
</tr>
<tr>
<td>VAT</td>
<td>Consumer expenditure</td>
</tr>
<tr>
<td>Fuel duties</td>
<td>Consumer expenditure</td>
</tr>
<tr>
<td>Excise duties</td>
<td>Consumer expenditure</td>
</tr>
<tr>
<td>Capital taxes</td>
<td>Equity and house prices and property transactions</td>
</tr>
<tr>
<td>Unemployment related expenditure</td>
<td>Unemployment</td>
</tr>
</tbody>
</table>

Elasticities of tax and expenditure bases with respect to the cycle

2.54 The sensitivity of each of the bases with respect to the output gap is estimated econometrically using the functional form in equation (8). This equation relates changes in a tax base, $X$, (e.g. wages and salaries) to changes in the contemporaneous and lagged output gap.
2.55 The variables wages and salaries, consumption and profits are expressed in terms of their ratio to potential output. The level of unemployment is expressed as the rate of unemployment consistent with potential output.\(^*\) For equity prices, house prices and property transactions we use estimates of their gap from equilibrium discussed in more detail in Chapter 4.\(^*\) The series are intended to represent a deviation from an estimated long run trend.\(^*\)

2.56 As before the output gap series used in this paper is taken from the OBR Working paper No. 1: Estimating the UK’s historical output gap as set out in Box 2.3. All the estimates are based on robust standard errors.

\[
\Delta \log(X_t) = \beta + \sum_{i=0}^{n} \alpha_i \Delta \log(Y_{t-i}/Y_{t-i}^*) + \nu_t
\]

2.57 This regression is estimated for each of the nine different bases identified in Table 2.16. The results are collated in Table 2.17. The coefficients on the output gap can be interpreted directly as the short-run elasticities of each tax and expenditure economic base with respect to the output gap. A coefficient greater than 1 (in absolute value) implies that swings in the economic cycle lead to the base moving by more than actual output.

---

\(^*\) The variable is the log(1-U/1-U*), where U* is the rate of unemployment consistent with potential output or NAIRU. For our estimates we use the OECD estimate of the NAIRU.

\(^*\) We use the so-called benchmark series for equity prices and the OECD house price gap series.

\(^*\) We use fiscal year data from 1972-73 to 2010-11 for the variables wages and salaries, consumption and non-oil, non-financial corporate profit. For financial company profits data is only available from 1982 and for the property transactions gap from 1978-79. The equity and house price gaps data is available from 1987-88.
Table 2.17: Elasticities of tax and expenditure bases with respect to the cycle

<table>
<thead>
<tr>
<th>Tax base</th>
<th>Output gap</th>
<th>Output gap (-1)</th>
<th>R Squared</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wages and salaries</td>
<td>0.73***</td>
<td>0.37</td>
<td></td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(0.20)^1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumer expenditure</td>
<td>1.14***</td>
<td>0.73</td>
<td></td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-oil, non-financial profits</td>
<td>4.16***</td>
<td>0.66</td>
<td></td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>(0.60)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Financial profits</td>
<td>1.18*</td>
<td>0.05</td>
<td></td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>(0.64)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equity prices (gap)</td>
<td>3.79</td>
<td>0.15</td>
<td></td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>(2.60)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>House prices (gap)</td>
<td>3.48*</td>
<td>0.13</td>
<td></td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>(1.77)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Property transactions (gap)</td>
<td>5.40*</td>
<td>0.23</td>
<td></td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>(3.00)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemployment</td>
<td>-3.77***</td>
<td>-3.34***</td>
<td>0.67</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>(0.74)</td>
<td>(0.62)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1 Robust standard errors in brackets

*** Significant at 1 per cent, ** Significant at 5 per cent, * Significant at 10 per cent. The equity price gap equation is significant at 20 per cent, if we exclude the last two years from the sample the variable becomes significant at 5 per cent, see Annex A.

2.58 The results in the table show the average response of each base to the cycle using the whole data sample. To check the robustness of the results we have also produced rolling window estimates which can be found in Annex A. The estimates indicate some variation in coefficients over time, especially during the economic cycle 1987-97.

Profit elasticity

2.59 The elasticity of profits to the cycle in Table 2.17 looks high compared to the coefficient on wages and salaries, although it is not inconsistent with what we would expect considering the current composition of profits and wages and salaries in output. However the resulting semi-elasticity of CT to the output gap, which is comparable to the one-step approach, is 0.1 slightly lower than the

21 A 1 per cent increase in nominal GDP in 2010-11 is higher, £ equivalent, than the combined increase in profits and wages and salaries suggested by the coefficient in Table 2.17. This is because the variables used in the econometric estimates, wages and salaries and profits, only account for around 65 per cent of GDP.
coefficient obtained from the one-step approach individual regression (Table 2.3).\textsuperscript{22}

2.60 Another way to estimate the elasticity of profits to the output gap is to make use of the National Accounting identity that national income is the sum of labour compensation (roughly speaking wages and salaries) and capital compensation (roughly profits). This implies that the elasticity of profits with respect to output must be proportional to the elasticity of wages and salaries with respect to output, as, loosely put, the two series sum to total output.\textsuperscript{23}

2.61 The estimate produced using this approach will depend on the assumption made of the share of profits in national income. If we assume that the profit share is around 24 per cent, which is consistent with National Accounts data in 2010, the elasticity is 1.9. If we however assume it is smaller or around 20 per cent of GDP, which is consistent with the variables used in this econometric estimate, the elasticity is 2.1. Using a simple metric of profits as share of profits and wages and salaries (around 29 per cent) gives us an elasticity of 1.6.

2.62 A lower profit elasticity such as the ones derived using this method might be preferred because the rolling window regression results, shown in Annex A, suggest that the sensitivity of profits to the cycle has diminished over time, while that of wages and salaries has increased. In Table 2.20 and Table 2.21 we show the sensitivity of the semi-elasticity to net borrowing using a profit elasticity of 1.6 and 2.1. Overall we would assume profits to be more sensitive to the business cycle than wages and salaries for example due to the stickiness of nominal wages.

Elasticities of tax receipts and expenditure with respect to the base

2.63 The sensitivity of each tax and expenditure category with respect to its base is estimated by the OECD using tax legislation and related fiscal data. The elasticity of personal income tax and social security contributions is, for example, estimated on the basis of statutory tax rates and the income distribution to which

\textsuperscript{22} Firstly we calculate the elasticity of CT with respect to the output gap, a combination of the elasticity of non-financial, non-oil profits with respect to the output gap (4.16) and the elasticity of CT receipts with respect to profits (1.5). This gives us 6.2. As non-oil non-financial CT is only around 2 per cent of GDP, this implies non-oil non-financial CT to GDP moves by around 0.1 per cent of GDP. See Annex B equation B2.

\textsuperscript{23} \frac{\Delta Z}{\Delta Y} \frac{Y}{Z} = \frac{\Delta Y - W + S}{\Delta Y} Y \left(1 - \frac{Z}{Y} \frac{\Delta W + S}{\Delta Y} Y \frac{Z}{W + S} Y \left(1 - \frac{Z}{Y}\right) e_{1.5}\right) here Z is profits.
they are applied. For corporate tax, indirect tax and unemployment benefits the OECD assumes an elasticity of unity.

2.64 We make use of ready reckoners produced by HMRC and DWP forecast models consistent with fiscal forecasts published in the OBR’s Economic and fiscal outlook (EFO) documents, when available and appropriate. Further information on the models can be found in the OBR’s Briefing paper No. 1: Forecasting the public finances and the ready reckoners are discussed in the OBR’s Briefing paper No. 4: How we present uncertainty, published alongside this paper. The results are shown in Table 2.18 and represent the impact of a 1 per cent increase in the relevant base.

Table 2.18: Elasticities of tax and expenditure categories to their bases

<table>
<thead>
<tr>
<th>Tax/expenditure category</th>
<th>Tax/expenditure base</th>
<th>Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income tax and NICs</td>
<td>Wages &amp; salaries</td>
<td>1.2</td>
</tr>
<tr>
<td>VAT</td>
<td>Consumer expenditure</td>
<td>1.0</td>
</tr>
<tr>
<td>Non-oil, non-fin corporation tax</td>
<td>Non-oil, non-fin corporation profit</td>
<td>1.5</td>
</tr>
<tr>
<td>Financial corporation tax</td>
<td>Fin corporate profit</td>
<td>1.5</td>
</tr>
<tr>
<td>Fuel duty</td>
<td>Consumer expenditure</td>
<td>1.0</td>
</tr>
<tr>
<td>Excise duties</td>
<td>Consumer expenditure</td>
<td>1.0</td>
</tr>
<tr>
<td>Business rates</td>
<td>Output</td>
<td>1.0</td>
</tr>
<tr>
<td>Capital gains tax</td>
<td>Housing</td>
<td>0.3</td>
</tr>
<tr>
<td>Inheritance tax</td>
<td>Housing</td>
<td>0.8</td>
</tr>
<tr>
<td>Stamp duty land tax</td>
<td>Housing</td>
<td>1.2</td>
</tr>
<tr>
<td>Stamp duty land tax</td>
<td>Transactions</td>
<td>1.0</td>
</tr>
<tr>
<td>Capital gains tax</td>
<td>Equity</td>
<td>1.8</td>
</tr>
<tr>
<td>Inheritance tax</td>
<td>Equity</td>
<td>0.5</td>
</tr>
<tr>
<td>Stamp duty shares</td>
<td>Equity</td>
<td>1.0</td>
</tr>
<tr>
<td>Unemployment related benefits</td>
<td>Unemployment</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Estimating the overall elasticity of the fiscal position to the cycle

2.65 The sensitivity of each tax receipts and expenditure item to the economic cycle can now be calculated by combining the estimates in the two previous sections. We present them in Table 2.19 both showing the elasticity in levels, or as a share of potential output, and as a share of GDP consistent with the one-step approach (see Annex B). For unemployment we combine the impact in year 1 and 2 for illustrative purposes.

24 For an individual worker the elasticity of income tax (social security contributions) with respect to income is calculated using the following equation:

$$\epsilon_{\text{worker}, w} = \sum_{i} \gamma_i MA_i / \sum_{i} \gamma_i AV_i$$

Here $\gamma_i$ is the weight of earnings group $i$ of total earnings, $MA_i$ is the marginal income tax rate at point $i$ (earnings group $i$) on the earnings distribution and $AV_i$ is the average income tax rate at point $i$ (earnings group $i$) on the earnings distribution.
Table 2.19: Elasticities of tax and expenditure categories to the output gap

<table>
<thead>
<tr>
<th>Tax/expenditure category</th>
<th>Elasticity level (sensitivity)</th>
<th>Elasticity share of GDP (semi-elasticity)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income tax</td>
<td>0.9</td>
<td>-0.01</td>
</tr>
<tr>
<td>NICs</td>
<td>0.9</td>
<td>-0.01</td>
</tr>
<tr>
<td>VAT</td>
<td>1.1</td>
<td>0.01</td>
</tr>
<tr>
<td>Non-oil, non-fin corporation tax</td>
<td>6.3</td>
<td>0.10</td>
</tr>
<tr>
<td>Financial corporation tax</td>
<td>1.8</td>
<td>&gt;0.01</td>
</tr>
<tr>
<td>Fuel duty</td>
<td>1.1</td>
<td>negligible</td>
</tr>
<tr>
<td>Excise duties</td>
<td>1.1</td>
<td>negligible</td>
</tr>
<tr>
<td>Business rates</td>
<td>1.0</td>
<td>0</td>
</tr>
<tr>
<td>Capital taxes</td>
<td>7.1</td>
<td>0.08</td>
</tr>
<tr>
<td>Spending (Unemployment)</td>
<td>-0.1</td>
<td>-0.44</td>
</tr>
</tbody>
</table>

2.66 To produce aggregated tax elasticity involves weighting the individual elasticities for each tax category by their share in total receipts. On this basis the total tax elasticity, in levels, is estimated to be 1.3 for year 1 and 0.0 for year 2. The total expenditure elasticity is estimated to be -0.04 for year 1 and -0.03 for year 2. This represents a change in tax and expenditure in response to movements in the output gap. This can be compared to the results from the one-step approach with minor adjustments (see more detail in Annex B). We show the results in Table 2.20.

2.67 The sensitivity of net borrowing to the cycle can then be measured by combining these two estimates as shown in equation (9). This is done by multiplying the elasticities with the ratios of tax and expenditure to GDP. The result tells us how much the level of net borrowing, or net borrowing as a share of potential GDP, changes with the cycle:

$$\sigma_{b,y} = \epsilon_{g,y} \left( \frac{G}{Y} \right) - \epsilon_{t,y} \left( \frac{T}{Y} \right)$$

2.68 To compare this to the results from the one-step approach we need to derive a semi-elasticity, which estimates how much net borrowing as a share of actual GDP responds to the cycle. This can be calculated using the following equation.

$$\tilde{\sigma}_{b,y} = (\epsilon_{g,y} - 1)\left( \frac{G}{Y} \right) - (\epsilon_{t,y} - 1)\left( \frac{T}{Y} \right)$$

25 Average share from 2001-02 to 2010-11, fiscal years
The difference between the two estimates is explained in more detail in Annex B. The semi-elasticity includes a term for borrowing as a share of GDP taking into account the denominator effect. Therefore if both the level of borrowing and the size of the output gap are small, the difference between the two estimates is small.

Table 2.20: Total net borrowing elasticity (negatives as for borrowing)

<table>
<thead>
<tr>
<th></th>
<th>Year 1</th>
<th>Year 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total tax (sensitivity)</td>
<td>1.30</td>
<td>0.00</td>
</tr>
<tr>
<td>Total expenditure (sensitivity)</td>
<td>-0.04</td>
<td>-0.03</td>
</tr>
<tr>
<td>Total tax (semi elasticity)</td>
<td>0.11</td>
<td>0.00</td>
</tr>
<tr>
<td>Total expenditure (semi-elasticity)</td>
<td>-0.43</td>
<td>-0.01</td>
</tr>
<tr>
<td>Net borrowing (sensitivity - equation 9)</td>
<td>-0.50</td>
<td>-0.01</td>
</tr>
<tr>
<td>Net borrowing (semi-elasticity - equation 10)</td>
<td>-0.54</td>
<td>-0.01</td>
</tr>
<tr>
<td>Net borrowing (semi-elasticity) profit elasticity (2.1)</td>
<td>-0.49</td>
<td>-0.01</td>
</tr>
<tr>
<td>Net borrowing (semi-elasticity) profit elasticity (1.6)</td>
<td>-0.47</td>
<td>-0.04</td>
</tr>
</tbody>
</table>

Overall, a one per cent change in the output gap is estimated to change net borrowing as a share of GDP by between 0.47 and 0.54 in the first year. This is consistent with our one-step estimate of 0.5 for the contemporaneous output gap set out in the first section of this chapter. However, the two-step econometric results alone suggest a small coefficient on the lagged output gap, compared to the 0.2 coefficient found in the one-step approach. This is partly due to the structure of the two-step approach which essentially imposes a non-lagged structure on the elasticity between the tax or expenditure item and its base. We consider this issue further in the next section.

The results from the one-step and two step approaches have limited sensitivity to different measures of the output gap (Box 2.3) and the alternative regression results reported in Annex A. For the two-step results, we have also investigated the effect of using different assumptions for the elasticity of profits with the respect to the output gap. Table 2.20 shows that it did not have a material impact on the results.
Box 2.3: The output gap

To cyclically adjust the public finances we generally need a prior estimate of the output gap. Such estimates will always remain highly uncertain since the level of potential output is never observed. The estimates also remain sensitive to the assumptions, data and methodology used. The OBR’s approach is to combine a range of indicators of the cyclical position of the economy.

Chart A: Estimates of the output gap

![Chart A: Estimates of the output gap](chart.png)

Different estimates of the output gap can produce different estimates of the elasticities of the budget balance with respect to the output gap. Previous Treasury estimates were produced using an alternative output gap series which was constructed using an on-trend point method. The OECD also produces their own estimates of the output gap to calculate their estimates of the UK’s cyclically adjusted budget balance.

To check the sensitivity of our estimates to different output gap measures we have re-estimated our results using the Treasury and OECD measures. The results for the two step approach are shown in Table A and indicate that the difference is relatively small overall. We also test the result from the one-step result using the old HMT series, which give similar results (see Annex A).

Table A: Cyclical adjustment coefficients

<table>
<thead>
<tr>
<th>Output gap</th>
<th>Year 1</th>
<th>Year 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>OBR basic model</td>
<td>0.5</td>
<td>0.0</td>
</tr>
<tr>
<td>OECD</td>
<td>0.4</td>
<td>0.1</td>
</tr>
<tr>
<td>HMT</td>
<td>0.4</td>
<td>0.1</td>
</tr>
</tbody>
</table>
Incorporating the effect of lags and a wider definition of spending

2.72 There are two types of lags in the way in which receipts and expenditure can respond to the cycle. First, the lag from movements in the output gap to movements in the relevant economic base, and second the lags from changes in the base to changes in receipts and expenditure.

2.73 The two-step approach estimates the first of these directly. The results suggest that the only base showing a lag to the output gap is unemployment, although this effect is found to be diminishing over time as shown in Annex A. The two-step approach does not directly take into account the second type of lag from the economic base to receipts and expenditure. Therefore to estimate this we have considered information on the structure of the UK tax and benefit system and corrected for the following:

- corporation tax (CT) has a one year collection lag for small businesses; and
- the final payment of self assessment (SA) and capital gains tax (CGT) liabilities is in the next financial year, i.e. a one year lag.

2.74 Adjusting for those suggests that around 0.1 of the estimated contemporaneous receipts elasticity would actually be lagged by one year. This gives us an overall elasticity of net borrowing to the output gap of 0.5 in the contemporaneous year and 0.1 in the lagged year.

2.75 As mentioned earlier the two-step approach only captures unemployment related spending and could therefore be underestimating the cyclicality of total spending. In particular, the results from the one-step approach show that looking at a wider measure of cyclical social security spending (including all DWP income related benefits and tax credits) rather than just unemployment, increases the coefficient on the lagged output gap by 0.1. Taking this into account suggests that we should add 0.1 to the lagged coefficient of the two-step result. The final result is therefore a contemporaneous coefficient of 0.5 and a lagged coefficient of 0.2.

Table 2.21: Total elasticity with lags and wider definition of spending

<table>
<thead>
<tr>
<th></th>
<th>Year 1</th>
<th>Year 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total tax</td>
<td>1.08</td>
<td>0.21</td>
</tr>
<tr>
<td>Total expenditure</td>
<td>-0.04</td>
<td>-0.03</td>
</tr>
<tr>
<td>Net borrowing (semi-elasticity)</td>
<td>-0.46</td>
<td>-0.09</td>
</tr>
<tr>
<td>Net borrowing (semi-elasticity) profit elasticity 2.1</td>
<td>-0.43</td>
<td>-0.07</td>
</tr>
<tr>
<td>Net borrowing (semi-elasticity) profit elasticity 1.6</td>
<td>-0.42</td>
<td>-0.07</td>
</tr>
<tr>
<td>Net borrowing (semi-elasticity) + cyclical social security</td>
<td>-0.46</td>
<td>-0.19</td>
</tr>
</tbody>
</table>
Conclusion

2.76 The pros and cons of each approach are summarised in Table 2.22. Overall we believe that the two-step approach has a number of advantages over the one-step approach.

2.77 Making use of HMRC and DWP’s detailed forecast models means the relationship between tax and expenditure items and their economic bases should be better estimated than in the one-step approach. Using the elasticities embedded in the models that the OBR uses to produce its fiscal forecast to calculate the cyclically-adjusted fiscal aggregates ensures there is an internal consistency to the calculations. These estimates should also represent the marginal impact consistent with the current tax and expenditure system. By contrast the one-step approach is essentially estimating the average relationship over the entire sample period.

2.78 By estimating the individual elasticities of a number of tax and expenditure items the two-step approach should also provide greater understanding of the contributions of individual items to the overall cyclical adjustment coefficients. The two-step approach also avoids the need to create a policy-adjusted tax series, which is difficult to construct and prone to measurement errors, making the econometric results more robust.

2.79 A potential drawback of the two-step approach is that it only captures unemployment-related spending. However, it is possible to correct for this by drawing on results from the one-step approach, as discussed above. Additionally, it doesn’t take into account wider cyclical factors that might affect receipts but not the relevant economic base. This is discussed further in Box 2.2.
Table 2.22: Comparing the approaches

<table>
<thead>
<tr>
<th>Pros</th>
<th>One-step</th>
<th>Two-step</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>Potentially captures other cyclical factors, like movement in tax evasion</td>
<td>-</td>
</tr>
<tr>
<td>-</td>
<td>Looks at a broader measure of spending</td>
<td></td>
</tr>
<tr>
<td>-</td>
<td>Method is simple and transparent</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cons</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>Has to rely on adjusted series that are difficult to estimate</td>
<td>-</td>
</tr>
<tr>
<td>-</td>
<td>The econometric estimates are therefore less robust</td>
<td>-</td>
</tr>
<tr>
<td>-</td>
<td>Estimates the average impact not the marginal impact</td>
<td></td>
</tr>
</tbody>
</table>

2.80 Table 2.23 summarises the results produced by the two approaches on a comparable basis (the transformations required to do this are explained in Annex B).

Table 2.23: Cyclical adjustment coefficients

<table>
<thead>
<tr>
<th>Approach</th>
<th>Year 1</th>
<th>Year 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>One-step</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>Two-step basic model</td>
<td>0.5</td>
<td>0.0</td>
</tr>
<tr>
<td>Two-step with lags</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>Two-steps with lags and adjustment to spending</td>
<td>0.5</td>
<td>0.2</td>
</tr>
</tbody>
</table>

2.81 The approaches produce broadly similar results. The adjusted results of the one-step approach produce cyclical adjustment coefficients of 0.5 in the first year and 0.2 in the second year. The two-step results produce 0.5 in the first year and 0.1 in the second year once the lags in the structure of the tax system are taken into account. Incorporating a wider measure of expenditure into the two-step approach produces coefficients of 0.5 and 0.2. We consider these coefficients to be appropriate for use in the OBR’s EFO forecasts, given our expectations that there are lags in the tax system and that cyclical expenditure extends beyond just unemployment related benefits.

2.82 Our results differ slightly from those of other institutions, discussed in Box 2.4, but the differences are relatively small. In practice, the estimate of the output gap used in the calculation of cyclically-adjusted deficits is more variable.
Box 2.4: Alternative cyclical adjustment estimates

The OECD regularly produces estimates of the elasticity of the current budget to the cycle for all its member countries, including the UK, using the two-step approach. The most recent UK estimate, from 2005, points to an overall elasticity of 0.45. This is slightly smaller than our central two step estimate of 0.5 in year 1 and 0.2 in year 2, which takes into account the lagged effect of a wider measure of cyclical social security.

The difference is probably due to the fact that the OECD uses a consistent method for all its member countries and that limits its ability to take into account specific features of tax and expenditure systems. We are able to allow for more disaggregation of tax receipts in our calculations, use information from HMRC and DWP forecast models, take into account lagged responses inherent in the UK tax system and use more econometric estimates instead of imposing unit elasticities. For example, the OECD assumes a unit elasticity of corporation tax receipts to profits for all countries. Using HMRC’s forecast model we assume an elasticity of 1.5.

The ECB produces a comparable estimate for EU countries, including the UK, set out in Bouthevillain et al (2001), but with a slightly different approach discussed in more detail in Chapter 3. The approach attempts to correct for changes in the composition of demand. The results point to a semi-elasticity of 0.65 in the first year, close to our estimate over two years. As with the OECD estimate, having to produce consistent estimates across the EU member states limits the ability to fit the calculations to each system. The main difference between the OECD and ECB results is the estimated response from income tax and unemployment, which the ECB finds to be larger.

° see Giouard and Andre (2005)

2.83 We can use the cyclical adjustment coefficients of 0.5 and 0.2 and the OBR historical output gap series to go back and estimate historical series of cyclical and structural net borrowing – Chart 2.2. For example, in the run up to the financial crisis in 2006-07 and 2007-08 structural net borrowing, adjusted for the cycle, is estimated to have been 2.7 and 3.5 per cent of GDP respectively. This is larger than previous HMT estimates of 2.3 and 2.6 per cent, see Chart 2.3, because of differences in output gap estimates. The structural position is also estimated to have been slightly higher in most of the 1990s. Given unchanged cyclical adjustment coefficients the OBR’s forecast for structural net borrowing would remain unchanged.
Chart 2.2: Structural and cyclical PSNB

Chart 2.3: Cyclical adjusted PSNB, previous and updated estimates

Source: OBR, HMT
3 Alternative approaches to cyclical adjustment

3.1 This chapter looks at two alternative approaches to the cyclical adjustment of the public finances. The first approach uses a more structural modelling approach, specifically a Structural Vector Autoregressive (SVAR) model, to allow for the influence of the fiscal position on the output gap. The second augments the two-step approach to allow for the effects of changes in the composition of demand on the cyclically-adjusted deficit.

SVAR model

3.2 The conventional approaches to cyclical adjustment, discussed in Chapter 2, require a prior estimate of the output gap. But this suffers from two main drawbacks (discussed in more detail in Annex C):

- first, it treats the estimation of the output gap and the cyclically-adjusted current balance as two separate events. It therefore ignores the possible feedback of fiscal policy to the output gap or reverse causality. If this feedback is present it can bias downward the estimated cyclical adjustment coefficients; and

- second, as Blanchard (1989) states, measures based on the output gap are ‘unnecessarily controversial’. There are many ways that the output gap can be calculated and the estimated cyclically-adjusted current budget can be sensitive to the choice of method adopted. This is discussed in more detail in Box 2.3.

3.3 This section explores an alternative approach to cyclical adjustment using a SVAR model. In this approach cyclical adjustment and estimation are tackled at the same time and no prior estimate of the output gap is required. This ‘direct’ approach therefore avoids some of the problems mentioned above.

3.4 For further detail on SVAR models see Annex D. In brief a vector auto-regressive (VAR) model is a system where each variable is regressed on its own lags and the lags of other variables in the system. A SVAR model is a form of VAR model where a certain structure has been imposed so that all the variables in the model are determined by the history of:
Alternative approaches to cyclical adjustment

- structural shocks (also known as impulses) - the trick is to give these economic meaning such as demand (business cycle) and supply (productivity) shocks; and
- the impact of these shocks on different variables in the model. This is characterised by the impulse response function which shows how each structural shock is propagated onto each variable.

Estimating the cyclically-adjusted current balance with an SVAR

3.5 In this section we present results for our estimates of the cyclically-adjusted current budget in the UK following an identification procedure similar to Blanchard and Quah (1989) using a two variable VAR.

3.6 As all variables in the unrestricted VAR are required to be stationary we estimate a two variable VAR consisting of:

- the growth rate of real GDP ($\Delta y$); and
- the current budget balance as a percentage of nominal GDP ($b / gdp$)

so that $Y = \begin{bmatrix} \Delta y \\ b / gdp \end{bmatrix}$

3.7 We stipulate two types of structural shocks:

- **productivity shocks**: these are supply shocks that only affect the trend growth rate of output; and
- **business cycle shocks**: these can be interpreted as temporary demand shocks and by definition have no long-run impact on GDP growth.

3.8 Specifying these two shocks and their long-run impact on the endogenous variables is sufficient to identify the SVAR model.

3.9 The basic model has the form:

$$Y = \mu + C^*(K)e_t, \quad e_t = \begin{bmatrix} e_{prod}^t \\ e_{cycle}^t \end{bmatrix}$$

(11)

Where $C^*(K)$ is a matrix of lag operators which describes the impulse response functions, i.e. how each of the structural shocks affects each of the model
variables; and \(e_t\) is the vector of shocks. The restrictions give the following long run representation:

\[
\begin{bmatrix}
\Delta y_t \\
\frac{b_t}{gdp_t}
\end{bmatrix} = 
\begin{bmatrix}
\mu_{\Delta y} \\
\mu_{b/gdp}
\end{bmatrix} + 
\begin{bmatrix}
\sum_{k=0}^{\infty} c_{11}(k) \\
\sum_{k=0}^{\infty} c_{21}(k) \\
\sum_{k=0}^{\infty} c_{22}(k)
\end{bmatrix} 
\begin{bmatrix}
e_{t}^{prod} \\
e_{t}^{cycle}
\end{bmatrix} \tag{12}
\]

3.10 Using lag selection criteria we estimate a two period lag for the SVAR model. The structural productivity and business cycle shocks are presented in Chart 3.1 and the impulse response functions are shown in Chart 3.2. The cyclically adjusted current budget can be produced by using the business cycle shocks and the impulse response of the current budget with respect to these shocks to deduce what the current budget would look like in the absence of these shocks. The results are presented in Chart 3.3.1

Chart 3.1: Structural productivity and business cycle shocks

Source: OBR

\[\text{1 In theory we could also construct an estimate of the trend productivity growth rate (and therefore the output gap) but this is not the focus of this exercise. In this SVAR approach 'productivity' is really a catch all type of supply side variable so population and capital may be included.}\]
Alternative approaches to cyclical adjustment

Chart 3.2: Impulse response of the current budget with respect to each structural shock

Alternative specifications

3.11 The modeller has a high degree of freedom to impose various forms of structure in the SVAR. In this section we consider two alternatives.

3.12 The first alternative (SVAR 2) replaces real GDP growth with the unemployment rate \( u_t \). The structural shocks in this model have the same interpretation as before but now demand shocks are identified by having no long run impact on the unemployment rate.\(^2\) Therefore the SVAR specification takes the following form:

\[
\begin{bmatrix}
  u_t \\
  b_t / gdp_t \\
\end{bmatrix}
= \begin{bmatrix}
  \mu_u \\
  \mu_{b / gdp} \\
\end{bmatrix}
+ \begin{bmatrix}
  \sum_{k=0}^{\infty} c_{11}(k) & 0 \\
  \sum_{k=0}^{\infty} c_{21}(k) & \sum_{k=0}^{\infty} c_{22}(k) \\
\end{bmatrix}
\begin{bmatrix}
  e^{prod}_t \\
  e^{cycle}_t \\
\end{bmatrix}
\]

(13)

\(^2\) Both variables are stationary according to unit root tests and lag selection criteria also suggest a second order VAR. We also considered a model using the change in the unemployment rate but this produced similar results. The SVAR could in principal be used to estimate the NAIRU by removing the effects of business cycle shocks from the unemployment rate.
3.13 As the unemployment rate and the real GDP growth rate tend to move together, a priori, we might expect there to be little differences between the two models.\(^3\) However this SVAR 2 model identifies larger business cycle and smaller productivity shocks than the previous specification.

3.14 The second alternative specification (SVAR 3) follows Hjelm (2003) in choosing a three variable model consisting of:

- the unemployment rate;
- the growth rate of real GDP; and
- the current budget balance as a percentage of GDP.

so that \[
Y = \begin{bmatrix}
u \\ \Delta y \\ b / gdp
\end{bmatrix}
\]

3.15 The SVAR representation is therefore driven by three shocks which we assume are:

- **labour market shocks**: these relate to factors like changes in social security systems, demography, hysteresis etc. that generate shifts in the vertical long-run Phillips curve (and hence long run aggregate supply). There are no restrictions imposed on this shock so it is allowed to affect the unemployment rate, real GDP growth and the current budget balance in the long run;

- **productivity shocks**: as before these are viewed as supply shocks, but while we assume that these can have a long run impact on real GDP growth and the current budget, we assume that there is no long-run impact on the unemployment rate. Following the rationale of the long-run Phillips curve the unemployment rate is determined only by the structure of the labour market; and

- **business cycle shocks**: as before these can be interpreted as temporary demand shocks that by definition have no long-run impact on the growth rate of real GDP (aggregate supply) or the unemployment rate (the long-run Phillips curve).

---

\(^3\) Rule of thumb estimates of the Okun’s Law for the UK suggest that the unemployment rate decreases by 0.5 to 0.75 ppt for a 1 per cent increase in the real GDP relative to potential GDP.
3.16 Specifying these three shocks and their long-run impact on the endogenous variables is sufficient to identify the SVAR model.

\[ Y = \mu + C^*(K)e_t \quad e_t = \begin{bmatrix} e_{LM}^t \\ e_{prod}^t \\ e_{cycle}^t \end{bmatrix} \]  

(14)

The restrictions give the following long run representation

\[
\begin{bmatrix}
    u_t \\
    \Delta y_t \\
    b / gdp_t
\end{bmatrix} = \begin{bmatrix}
    \mu_u \\
    \mu_y \\
    \mu_{b/gdp}
\end{bmatrix} + \begin{bmatrix}
    \sum_{k=0}^{\infty} c_{11}(k) \\
    \sum_{k=0}^{\infty} c_{31}(k) \\
    \sum_{k=0}^{\infty} c_{33}(k)
\end{bmatrix} \begin{bmatrix}
    0 \\
    \sum_{k=0}^{\infty} c_{22}(k) \\
    \sum_{k=0}^{\infty} c_{32}(k) \\
    \sum_{k=0}^{\infty} c_{33}(k)
\end{bmatrix} e_{LM}^t + \begin{bmatrix}
    0 \\
    0 \\
    \sum_{k=0}^{\infty} c_{33}(k)
\end{bmatrix} e_{prod}^t + \begin{bmatrix}
    0 \\
    0 \\
    \sum_{k=0}^{\infty} c_{33}(k)
\end{bmatrix} e_{cycle}^t
\]

(15)

3.17 Here the business cycle shock is identified by having no long run impact on the unemployment rate and real GDP growth rate, and the productivity shock is identified by having no long run effect on the unemployment rate.  

3.18 Comparisons of the three cyclically-adjusted current budget estimates are presented in Chart 3.3. The differences highlight the relative size and impact of the estimated demand shocks in each set up. In the first alternative model (SVAR 2) these demand shocks are estimated to have a greater impact on the current budget resulting in a larger cyclical adjustment. In the second alternative (SVAR 3), there are two sources of supply shocks – labour market and productivity – which likely reduces the contribution of business cycle shocks to the dynamics of the current budget and leads to a smaller cyclical adjustment.

---

4 All variables in the VAR are stationary and lag selection criteria suggest a second order VAR. The model was also estimated using the change in the unemployment rate and produced similar results.
Issues with SVAR models

3.19 SVAR models have two main advantages. First they do not require a prior estimate of the output gap. Second, they address the endogeneity between output (gap) and the current budget that might otherwise lead to simultaneous equations bias. However, there are also a number of objections to the SVAR methodology.

3.20 The relatively small number of variables in VAR models means that the entire system typically ends up being driven by a small group of fundamental shocks such as demand and supply. The effects of other relevant shocks are not fully captured in the model because they don’t fit well into these classifications. Furthermore, defining shocks as either demand or supply can be perilous.

3.21 As judgements are required to identify the SVAR system, Uhlig (1999) points out the possible degree of circularity in SVAR conclusions. Assumptions imposed in previous work, which have not been empirically tested, may be used to justify

---

5 Because a VAR model is estimated as a system rather than a number of individual equations, the number of coefficients that need to be estimated grows quickly as the number of variables in the model increases. As a result the number of degrees of freedom available in the estimation run out much faster which usually means VAR models are restricted to low dimensions (i.e. a small number of variables). Here the problem is compounded by the data being low frequency (fiscal year) which reduces the number of possible observations/degrees of freedom available.
imposing a particular set of restrictions. The dynamics of the model can therefore reflect the prejudice of the modeller as much as the data and informal restrictions in the role of identifying shocks lead to undisciplined data mining. These issues make it difficult in practice to use this method for forecast purposes.

**Component method (ECB)**

3.22 The fiscal position is likely to be sensitive to changes in the composition of demand. Domestic demand led growth is, for example, likely to affect tax revenue more than growth led by the export industry. The ECB approach attempts to correct for such output composition effects by using separate estimates of cyclical components of individual tax and expenditure bases.\(^6\)

3.23 As with the two-step approach discussed in Chapter 2, the ECB approach is disaggregated - tax revenue is broken down into individual tax categories and the relevant tax bases are identified. On the expenditure side unemployment related benefits are considered to be cyclical and move with changes in the unemployment gap.

**Overview of method**

3.24 Cyclically-adjusted net borrowing, \( b^* \), can be defined as the difference between net borrowing (\( b \)) and the cyclical component of borrowing (\( b_c \)):

\[
b^* = b - b_c = b - \sum_{i=1}^{n} b_c^i
\]

(16)

3.25 Furthermore, \( b_c^i \) can be defined as the cyclical component of each budget category \( i \). To compute \( b_c^i \), in line with the ECB approach, a trend series for each of the tax and expenditure bases is constructed. The trend series is used to estimate a gap series which represents the deviation of the base (macro-economic variable) from its equilibrium trend level. The gap series are analogues to the output gap, which is a gap series for output. In addition to the gap series an estimate for the elasticity of the tax receipts and expenditure to the relevant base (\( \varepsilon_{b_c^i} \)) is estimated.

\[
b_c^i = b^i \varepsilon_{b_c^i} v_c^i
\]

(17)

\(^6\) Bouthevillain et.al. (2001).
Alternative approaches to cyclical adjustment

3.26 Here \( V^i_t \) is the gap of the corresponding macro-economic variable \( V \) in real terms from its trend level. After substituting equation 17 in to equation 16 the cyclical balance in year \( t \) can be defined as:

\[
\begin{align*}
    b^*_t &= b_t - \sum_{i=1}^{n} b^i \varepsilon_{b^i V^i_t} V^i_t \\
    &\text{(18)}
\end{align*}
\]

3.27 This method does not produce cyclical adjustment coefficients comparable to those produced with the approaches discussed in Chapter 2 and presented in Table 2.23. This is because the correction in relation to the output gap varies over time since the relationship between the various gap series and the output gap is not constant over time.

Elasticities of tax and expenditure with respect to their base

3.28 In the ECB approach the sensitivity of the tax receipts and expenditure to their bases are calculated by using an econometric regression or derived from tax or expenditure regulation similar to the OECD approach described in Chapter 2. In this paper we make use of the existing forecast tools used in the OBR’s economic and fiscal forecast to assess the relationship between individual tax and expenditure categories and their relevant bases (see Table 2.18).

The gap estimates

3.29 In line with Bouthevillain et al. (2001) we use Hodrick-Prescott (HP) filters with a smoothing parameter of 30 to estimate the gap measures for each of the following tax and expenditure bases: wages and salaries, non-oil, non-financial profits, financial profits, consumption and unemployment. The gap refers to the deviation of the variable from its equilibrium growth path. To correct for the end point problem we extend the series using the OBR’s latest forecast and an AR(4) model. The gap estimates for wages and salaries and consumer expenditure are shown in Chart 3.4. To deflate the series we use the consumption and GDP deflators.
Alternative approaches to cyclical adjustment

Chart 3.4: Wages & salaries, consumer spending and output gaps

To understand the relationship between the gap measures it is useful to look at correlations over time. If the gap measures differ markedly from movements in the output gap it suggests an output composition effect. The correlation results in Table 3.1 indicate that a composition effect could be present.

Table 3.1: Correlation with the output gap

<table>
<thead>
<tr>
<th></th>
<th>Wages &amp; salaries gap</th>
<th>Consumption gap</th>
<th>Non-oil, non-fin. profits gap</th>
<th>Financial profits gap</th>
<th>Unemployment gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output gap</td>
<td>0.3</td>
<td>0.6</td>
<td>0.7</td>
<td>0.4</td>
<td>-0.7</td>
</tr>
</tbody>
</table>

Re-estimating the structural deficit

Combining the gap estimates and the elasticities from Chapter 2 we can re-estimate the structural deficit taking into account composition effects by using equation (18). The difference between the results and the latest cyclically-adjusted current budget (CACB) estimate is quite large, especially in 2007-08. The main reason is that wages and salaries, an important variable for tax receipts, has a low correlation with the output gap in recent years reflecting the relatively strong performance of the labour market compared to the size of the fall in output.
Alternative approaches to cyclical adjustment

Table 3.2: Re-estimated cyclically adjusted current budget

<table>
<thead>
<tr>
<th>Share of GDP</th>
<th>Current budget</th>
<th>CACB (Composition)</th>
<th>CACB (Composition with adjustment for capital taxes)</th>
<th>CACB (March 2012 EFO)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006-07</td>
<td>-0.4</td>
<td>-1.0</td>
<td>-1.1</td>
<td>-0.4</td>
</tr>
<tr>
<td>2007-08</td>
<td>-0.3</td>
<td>-1.7</td>
<td>-1.9</td>
<td>-0.6</td>
</tr>
<tr>
<td>2008-09</td>
<td>-3.5</td>
<td>-3.8</td>
<td>-3.6</td>
<td>-3.1</td>
</tr>
<tr>
<td>2009-10</td>
<td>-7.7</td>
<td>-7.3</td>
<td>-7.0</td>
<td>-5.5</td>
</tr>
</tbody>
</table>

3.32 Unlike the estimates in Chapter 2 this approach does not provide a straightforward way to incorporate cyclical adjustment to capital tax receipts since they are not directly related to movements in output or output composition. One way to incorporate their effect would be to construct ‘cyclical’ asset price and transaction gap series consistent with the results from the econometric estimate in Chapter 2. That tells us how much those gap series move with the cycle. We do the opposite in Chapter 4 when we use cyclically adjusted asset price and transaction gap series, the residual, to adjust for fluctuations unrelated to the cycle. The impact from capital taxes using this method, shown in Table 3.2, is found to be relatively small.

3.33 The main benefit of the ECB approach is that it attempts to correct for the composition of demand which could give a better picture of the structural position. The main disadvantage is that it relies on a simple estimate of a number of gap series which can be prone to measurement error. This would be particularly challenging for forecast purposes since the method applied – a Hodrick Prescott (HP) filter- implies an end point problem where the estimates are less reliable at the end of the data series – the area often of most interest. In addition, unlike the methods in Chapter 2, it does not produce a single cyclical adjustment coefficient relating borrowing to the output gap and therefore is less transparent. It cannot also easily take into account the cyclicality of capital taxes.

3.34 The ECB approach looks at the effect of changes in composition of demand, but it does not explicitly consider the effect of changes in the relative prices of the demand components. One further possible refinement of our cyclical adjustment calculations is to allow for the potential effect of changes in the terms of trade. We consider this issue briefly in Box 3.1.
Box 3.1: Adjusting for the terms of trade

If the fiscal balance depends heavily on revenue from commodity exports or changes in the terms of trade, a correction for movements in relative prices could give a better idea of the underlying structural balance. A simple way to do this is to look at movements in the real income gap instead of the output gap (Turner (2006)).

Real gross domestic income measures the purchasing power of total income from domestic production which includes gains from terms of trade changes. The difference between gross domestic income and gross domestic product is therefore equal to those gains. The income gap (IG) is equal to the output gap (OG) plus the terms of trade gap times the share of exports in GDP (EX).

\[ IG = OG + EX(TT - TT^*) \]

The terms of trade gap can be estimated using a Hodrick Prescott (HP) filter similar to the calculations in the ECB approach in Chapter 3. The gap for the UK is the difference between the series in Chart A.

Chart A: Income and output gap

Having estimated the terms of trade gap we can calculate the income gap. High correlation between the income and output gap measures, as shown on Chart A, indicates little need to correct for movements in the terms of trade for the UK.

To calculate the effect of changes in the terms of trade we re-estimated the elasticities of the tax and spending bases with respect to the output gap by including the component of the terms of trade gap. This should give us the response to the real
Alternative approaches to cyclical adjustment

income gap split into the output gap and the terms of trade gap. The following equation was estimated for each base (X).

\[ \Delta \log(\frac{X_i}{Y_i^*}) = \beta + \sum_{i=0}^{n} \alpha_i \Delta \log(\frac{Y_{i-1}}{Y_{i-1}^*}) + \sum_{t=0}^{m} \delta_i \Delta \log(\frac{EX(T_{i-t}}{T_{i-t}^*})) + \nu_i \]

The terms of trade gap component is shown to affect wages and salaries, consumption and house prices in year 1. The elasticity of the terms of trade gap component to tax receipts as a share of GDP is calculated to be around 0.35. However, since the terms of trade component (the difference between the two series in Chart A) has historically been very small this would amount to an insignificant correction to the fiscal position.

Conclusion

3.35 Chart 3.5 shows estimates of the cyclically adjusted current budget using the preferred cyclical adjustment coefficients, the OBR’s output gap series from Chapter 2 and the main results from the approaches presented in this Chapter. The main SVAR result is smoother than the other two while the ECB result, excluding capital tax adjustment, indicates a larger structural deficit in recent years and for the forecast period.

Chart 3.5: Estimates of the CACB

![Chart 3.5: Estimates of the CACB](image-url)
3.36 The approaches are designed specifically to address some of the perceived weaknesses of the more standard approaches discussed in Chapter 2.

3.37 The SVAR approach sidesteps the requirement to make an estimate of the output gap and can take into account any feedback from fiscal policy on the economic cycle. However, the identifying procedure is subjective and the accuracy of the estimates largely depends on the restrictions imposed.

3.38 The ECB method is designed to allow for the possibility that shifts in the composition of growth between more or less ‘tax-rich’ components of aggregate demand can be important for the fiscal position. The results emphasise the importance of the composition of growth to the fiscal forecast. This is something the OBR will continue to monitor and analyse as a risk to the fiscal forecast in future EFOs. However the component gap series require an estimate of the benchmark composition of demand. But, unlike potential output, there is no equivalent reference for the equilibrium structure of aggregate demand. Furthermore the method is not as robust for forecast purposes since the simple gap estimates are less reliable at the end of the data series.
4 Asset price and transaction adjustments

4.1 The approaches discussed in the preceding chapters attempt to adjust the fiscal position for the effects of the economic cycle on government receipts and expenditure in order to produce an estimate of the underlying or ‘structural’ fiscal position. However, the underlying fiscal position can also be affected by other temporary factors that do not necessarily move in line with the cycle, such as movements in commodity and asset prices.

4.2 This chapter therefore considers whether it is possible to adjust the fiscal position to take account of fluctuations in asset market prices and transactions. In particular our focus is on the effect of prices and transactions in the housing and equity markets on receipts from capital taxes. We also extend the analysis to consider whether asset market movements have driven the level of receipts from a wider set of taxes, including income tax, VAT and corporation tax.

4.3 This issue is particularly relevant in the UK given the size of the financial sector relative to GDP and relatively high levels of owner-occupation of housing compared to other countries. One of the key factors driving the deterioration in the UK public finances following the financial crisis in 2008 was a sharp fall in receipts from the housing and financial sectors. An understanding of how the level of prices and transactions in asset markets is affecting tax receipts is therefore important to assessing the sustainability of the UK fiscal position.

Adjusting for asset prices and property transactions

Overview of method

4.4 To estimate the degree to which asset markets may temporarily affect the underlying fiscal position it is first necessary to assess the position of prices and transactions in asset markets compared to some concept of their equilibrium level. We first discuss approaches which have been used to make such an assessment, and the uncertainties involved.

4.5 We then assess the potential impact on tax receipts of the estimated deviation of asset markets from the equilibrium level using both the one-step and two-step approaches from Chapter 2.
Estimating asset-price gaps

4.6 Estimating the deviation of asset prices from their equilibrium level is the key aspect of this analysis. However, this is far from straightforward. Indeed, the very concept of asset markets deviating from equilibrium is itself contentious, particularly in more liquid markets such as those for equities. The efficient market hypothesis would imply that asset prices in liquid markets adjust to publicly available information very rapidly, and therefore should always reflect the fundamental value. Any deviation from the equilibrium would lead to the asset being perceived as valued incorrectly. There would then be an incentive for market participants to trade until perceived misalignments and arbitrage opportunities were eliminated. Under this theory asset price movements will always be the result of new information, for example about financial innovation, demographic change and the expected growth in real incomes.

4.7 However, a more widely accepted view is that deviations from fundamental values may persist for reasons relating to information or behaviour of market participants. For example, information asymmetries, cognitive bias, herd behaviour and financial market frictions have all been cited as causes of such movement. Although ultimately temporary, these deviations may be sustained for periods that could be significant for the public finances. The effect on tax receipts of movements of longer-term or fundamental components of asset prices would be structural, while revenues attributable to the deviation of prices from those fundamentals would be temporary.

4.8 The OBR’s forecasts for asset markets contained in the *Economic and fiscal outlook* are based on conditioning assumptions. The equity market is assumed to grow in line with nominal GDP implying a constant equity price to company earnings (P/E) ratio. The housing market forecast is initially the median expectation of the *Treasury’s Comparison of Independent Forecasts*, and thereafter rises in line with the rate of earnings growth. These approaches are not based on concepts of an explicit equilibrium asset price, which limits the scope for applying the analysis in this chapter to the EFO forecast.

4.9 There is no consensus in the wider literature on the best method for measuring equilibrium asset prices and therefore estimating deviations from the equilibrium. For example, Jaeger and Schucknecht (2004) use a technique to locate asset price booms and busts that determines turning points in this series. A more mechanical approach is taken by Girouard and Price (2004), who attempt to distinguish the cyclical element of asset price movements by using a Hodrick-Prescott filter.

4.10 In its 2008 working paper HM Treasury used an approach based on defining a historical benchmark level for asset prices. It defined the housing price...
benchmark as the observed median value of the ratio of real house prices to real
disposable income per capita, and used the median ratio of share prices to
nominal GDP to define the share price benchmark. It then compared actual
prices to these benchmark levels to estimate deviations from equilibrium. We
have updated and replicated this approach in this paper. In more recent work for
the OECD, Dang and Price (2011) attempt to adjust fiscal balances for asset
price cycles according to a more fundamentals-based approach. We have also
used this approach in this paper and the gap series we have derived are
discussed in more detail below.

Equity price gaps

4.11 For equities, Dang and Price (2011) use a form of the Gordon equity pricing
formula to estimate values for fundamental equity prices. The formula states that
in the long run the dividend yield plus the future growth in earnings should
correspond to the risk-free interest rate plus a risk premium. By substituting
corporate earnings growth for the growth rate in dividends the steady state
relationship should be:

\[
\frac{P}{E}^* = \frac{(1+g)}{(r+\sigma-g)}
\]

where \( r \) is the risk-free interest rate, \( \sigma \) is the risk premium, and \( g \) is the long-run
growth of earnings. This leaves \( P/E^* \) as the equilibrium or ‘expected’ measure of
the price/earnings ratio. Certain proxies have to be used in the construction of
this estimate. The risk-free interest rate is based on the nominal 10 year
government bond yield, plus the spread between the US AAA corporate yield and
the US Treasury 10-year bond yield. Long-run corporate earnings growth is
constrained to equal the year-on-year growth rate of the economy.

4.12 Following the OECD approach, the risk premium is set at 4 per cent. However,
as with its two-step approach to cyclical adjustment, the OECD’s choice here is
partly motivated by simplicity and comparability across nations. In reality the risk
premium is unlikely to be constant, and will change over time with attendant
economic factors.

House price gaps

4.13 For house prices, Dang and Price (2011) use a housing valuation model based
on Poterba (1984) where long-term equilibrium house prices are determined by
the influence of the user cost of housing on the price-to-rent ratio. In equilibrium
the model shows rents equal to the user cost:

\[
\frac{P}{R}^* = \frac{1}{i_u + \tau + f - \pi}
\]
where $i_a$ is the after-tax nominal mortgage interest rate, $\tau$ the property tax rate on owner-occupied houses, $f$ the recurring holding costs consisting of depreciation, maintenance and the risk premium on residential property and $\pi$ the expected capital gain on houses. The mortgage rate is proxied by using 10 year government bond yields. The expected capital gain is approximated by a five-year moving-average of consumer price inflation. Depreciation, maintenance costs and the risk premium are assumed to be 4 per cent.

Estimating residential property transactions gap

4.14 Cycles in the housing market are associated not only with movements in house prices but also with movements in the level of property transactions. A sharp fall in house prices, relative to trend, could have a larger effect on tax receipts if it is associated with a fall in transactions.

4.15 To construct a gap series for property transactions we firstly need to estimate an equilibrium trend level of transactions. We make a simple assumption that the trend level is consistent with the average level of owner occupied duration over time. We calculate the average implied duration ($D_u$) by dividing the housing stock ($H$) with property transactions ($PD$)\(^1\), equation 21. We find the average duration to be around 19 years.

\[
\frac{\sum_{t=0}^{N} H_t}{\sum_{t=0}^{N} PD_t} = D_u
\]  

(21)

4.16 The trend level of transactions is then simply calculated as the housing stock divided by average duration:

\[
\overline{PD} = \frac{H}{D_u}
\]  

(22)

Results using the benchmark and gap approaches

4.17 Charts 4.1 and 4.2 show the results for these series using both the OECD gap approach and the benchmark approach. The benchmark for equity prices is the observed median value of the ratio of share prices, using the FTSE all share, to nominal GDP. The house price gap differs from the previous HMT method, by

\(^1\) Data on housing stock is from the Department for Communities and Local Government and data on property transactions from HMRC
using the ratio of real house prices to real rents, rather than the HM Treasury approach using the ratio of real house prices to real disposable income per capita. As can be seen in Chart 4.1, using this approach the benchmark series closely follows the OECD approach.

4.18 Both charts show the estimated percentage point deviation of the asset price from the estimated equilibrium value. Under both approaches the level of estimated deviation from equilibrium is very volatile, sometimes reaching up to 50 per cent above or below the estimated equilibrium.

4.19 The two equity price series evolve in very different ways over this time period. For example, the OECD approach suggests that equities were around 50 per cent undervalued in the run-up to the financial crisis in 2008 which does not seem particularly plausible. By contrast the benchmark approach suggests equities were close to their equilibrium value in 2007-08 and then undervalued by around 16 per cent of GDP by 2009-10. The choice of equity risk premium in the OECD approach can affect the series quite significantly, which is a potential disadvantage of this approach.

4.20 Historic movements in both the OECD and benchmark methods for estimating house prices gaps are more similar, but there is some divergence over recent periods. The OECD approach suggests house prices have fallen toward their equilibrium value since the financial crisis, which seems reasonably plausible, while the benchmark approach suggests they remain significantly overvalued. This may be explained by the historically low interest rates following the financial crisis, which will not be picked up in the benchmark approach. This may suggest the OECD approach is better suited for analysis of house prices. The analysis suggests that transactions fell significantly below their equilibrium level after the financial crisis which is in line with expectations.

4.21 In neither approach is there an obviously strong correlation between the asset price series and the output gap. This suggests that asset price fluctuations may not be captured by the cyclical adjustment coefficients estimated in Chapter 2. However, the significant differences between the series generated using the two approaches highlights the difficulties involved in generating a robust estimate of asset price gaps.
Asset price and transaction adjustments

Chart 4.1: House price and transactions gaps

[Graph showing house price and transactions gaps]

Source: OBR, OECD

Chart 4.2: Equity price gaps

[Graph showing equity price gaps]

Source: OBR, OECD
Measuring the effect on receipts

4.22 Having derived measures of the deviation from a normal or equilibrium level, we then assess the potential impact on tax receipts of the estimated deviation of asset markets from their equilibrium levels using the one-step and two-step approaches from Chapter 2. For our central estimate we choose a hybrid combination of asset price gap series which we believe to be most plausible by using the benchmark equity price gap series and the OECD house price gap series.

One-step approach

4.23 If asset prices move in step with the output gap then the one-step approach discussed in Chapter 2, which we have extended to cover capital taxes, should already fully capture their effect on the public finances. But the divergence between the asset price gaps and the output gap shown in Charts 4.1 and 4.2 suggests that movements in asset prices may be subject to somewhat longer ‘cyclical’ swings than standard measures of the business cycle. Some of their effect may therefore be missed by standard approaches. We can investigate this proposition more formally by regressing the cyclically-adjusted tax receipts series derived in Chapter 2 on these asset price gap series. Once again replicating the previous Treasury analysis, we estimate the following regression:

\[
CATR_t = \alpha_1 CATR_{t-1} + \alpha_2 Egap_t + \alpha_3 Hgap_t + \alpha_4 Tgap_t
\]

where CATR is cyclically-adjusted tax receipts, Egap is the equity price gap, Hgap is the house price gap and Tgap is the transaction gap. The sample period begins in 1987-88 due to a lack of data availability and ends in 2006-07 for the Treasury results, and 2010-11 for our updated estimates. For the ‘hybrid’ combination of the benchmark approach for equity prices and the OECD house price gap series, we extend the analysis to include the effect of the transactions gap shown in Chart 4.1. The results are shown in Table 4.1.

---

2 Calculated using the tax coefficients derived in the one-step approach (0.1 x contemporaneous output gap + 0.1 x lagged output gap) and the OBR’s estimate of the output gap. Note that this calculation is applied to total tax receipts, not just the subset of receipts included in the regression.

3 Note that as discussed in paragraph 4.19 our definition of the benchmark house price gap differs from the previous Treasury definition.
Asset price and transaction adjustments

Table 4.1: Asset price and transaction effects

<table>
<thead>
<tr>
<th></th>
<th>Constant</th>
<th>CATR (-1)</th>
<th>Equity price gap</th>
<th>House price gap</th>
<th>Transactions gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>HMT 2008 (Benchmark)</td>
<td>0.47</td>
<td>0.025</td>
<td>0.019</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OBR (Benchmark)</td>
<td>11.7***</td>
<td>0.68***</td>
<td>0.034***</td>
<td>0.014**</td>
<td></td>
</tr>
<tr>
<td>OBR (OECD)</td>
<td>8.0**</td>
<td>0.78***</td>
<td>-0.001</td>
<td>-0.001</td>
<td></td>
</tr>
<tr>
<td>OBR (Hybrid)</td>
<td>9.5***</td>
<td>0.74***</td>
<td>0.030***</td>
<td>0.011*</td>
<td></td>
</tr>
<tr>
<td>OBR (Hybrid &amp; transactions)</td>
<td>15.4***</td>
<td>0.59***</td>
<td>0.024**</td>
<td>0.011*</td>
<td>0.021**</td>
</tr>
</tbody>
</table>

*** Significant at 1 per cent level; ** Significant at 5 per cent level; * Significant at 15 per cent level.

4.24 The Treasury’s 2008 results implied that a 10 per ‘over-valuation’ of both equity and house prices would boost cyclically-adjusted tax receipts by around 0.44 per cent of GDP. 4 We find similar results for the benchmark approach, which suggest a slightly larger adjustment of 0.48 per cent of GDP, with a little more of the adjustment attributable to equity price fluctuations. The results using the OECD based fundamentals approach show no economic or statistical significance. The results using our preferred ‘hybrid’ approach yield similar results to the benchmark approach but with slightly more limited statistical significance. The addition of the transactions gap to the hybrid equation marginally reduces the importance of the equity gap, but boosts the overall size of the asset adjustment.

4.25 Chart 4.3 shows the additional adjustments that would need to be made to the cyclically-adjusted receipts to adjust for asset price effects using this ‘hybrid’ and transactions approach.

---

4 The results shown here were the Treasury’s preferred estimates and are median values of the reported results across a range of sample periods. The results are also the same as the point estimates for the sample period starting in 1987-88.
The results appear reasonably intuitive. On this analysis, temporary buoyancy in asset prices may have overstated the strength of the structural fiscal position by 1 per cent of GDP in 1999-00, around the time of the ‘dot com’ boom. House price and transaction effects have a similar effect in the run-up to the financial crisis, also boosting the apparent strength of the structural balance by 1 per cent of GDP in 2006-07 (with some help from equity prices). This analysis suggests that over 2 per cent of the deterioration in the fiscal position between 2006-07 and 2008-09 relates to a sharp fall in asset prices and housing transactions.

Two-step approach

The next section considers the issue of asset price adjustment in the context of the two-step approach. We use, as before, ready reckoners of the sensitivity of receipts to prices and housing transactions produced by HMRC, as shown in Table 4.2. These ready reckoners can then be applied to the estimated deviation in the asset price from the ‘equilibrium’ level to estimate the size of the revenue gain or loss from the divergence. The ready reckoners suggest that asset price and transactions mainly influence capital taxes.

The elasticities shown are calculated for the OBR’s latest forecast, and in this sense do not show the historic relationship that may have existed in the past. In effect, they show the change in receipts under current policies. We correct the asset price and transaction gap series for cyclicality using the results from Chapter 2 where we estimated how much the series move with the output gap.
Asset price and transaction adjustments

We do this to make sure we are capturing other fluctuations not related to the cycle.

Table 4.2: Elasticity of tax receipts to asset prices and transactions

<table>
<thead>
<tr>
<th>Tax/spending category</th>
<th>Asset</th>
<th>Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital gains tax</td>
<td>Housing</td>
<td>0.3</td>
</tr>
<tr>
<td>Inheritance tax</td>
<td>Housing</td>
<td>0.8</td>
</tr>
<tr>
<td>Stamp duty land tax</td>
<td>Housing</td>
<td>1.2</td>
</tr>
<tr>
<td>Capital gains tax</td>
<td>Equity</td>
<td>1.8</td>
</tr>
<tr>
<td>Inheritance tax</td>
<td>Equity</td>
<td>0.5</td>
</tr>
<tr>
<td>Stamp duty shares</td>
<td>Equity</td>
<td>1.0</td>
</tr>
<tr>
<td>Stamp duty land tax</td>
<td>Transactions</td>
<td>1.0</td>
</tr>
</tbody>
</table>

4.29 These results, using our preferred gap series, suggest that fluctuations in asset markets have led to temporary deviations in capital tax revenue of close to 0.4 per cent of GDP. For example, in the run up to the financial crisis capital tax receipts were potentially above their ‘structural’ level by around 0.3 per cent of GDP. The bulk of this is due to the estimate that house prices were overvalued in this period. Equity prices were estimated to be close to equilibrium value at this time using the benchmark approach.

4.30 The estimated deviation in revenue in the run-up to the financial crisis looks small in comparison to the deterioration in total tax receipts seen after the crisis. This is partly because the approaches we have used suggest equity prices were not overvalued in this period. In addition, however, capital taxes only represent a small share of total tax. Therefore in the next section we extend the two-step approach to consider the potential effect of asset price and transactions gaps on a wider set of taxes.

Two-step extended to other taxes

4.31 Movement in asset prices and transactions could also affect other types of taxes. For example, higher asset prices might drive higher wages in the financial sector, especially bonus payments, which could in turn boost income tax receipts. Higher asset prices and increased volumes of property transactions could also have created a wealth effect, causing an increase in consumption and therefore VAT and excise duty receipts. Higher asset prices could also boost corporate profits, especially in the financial sector, and increase corporation tax receipts.

4.32 To investigate whether asset price and transactions gaps affect other tax receipts we re-estimate the equations in Chapter 2. We modify the equations by adding the additional gap series as explanatory variables. We use the same preferred measures of equity and house price gaps as before.
\[ \Delta \log \left( \frac{X_i}{Y_i} \right) = \beta + \sum_{i=0}^{n} \alpha_i \Delta \log \left( \frac{Y_{i-1}}{Y_i} \right) + \sum_{i=0}^{n} \delta_i \Delta \log (Egap_{i-1}) + \sum_{i=0}^{n} \delta_i \Delta \log (Hgap_{i-1}) + \sum_{i=0}^{n} \delta_i \Delta \log (trans_{i-1}) + \nu_i \]  

4.33 The results suggest that the equity price gap affects wages and salaries, consumption and financial sector profits while the transactions gap is found to affect consumption. The house price gap is not found to significantly influence any of the other tax bases.

### Table 4.3: The elasticity of the tax base with respect to various gap measures

<table>
<thead>
<tr>
<th>Tax base</th>
<th>Output gap</th>
<th>Equity price gap</th>
<th>Transaction Gap</th>
<th>R squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wages and salaries</td>
<td>0.35*</td>
<td>0.03*</td>
<td></td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>(0.20)</td>
<td>(0.02)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumer expenditure</td>
<td>0.77***</td>
<td>0.04***</td>
<td>0.02*</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td></td>
</tr>
<tr>
<td>Fin profits</td>
<td>1.41</td>
<td>0.27*</td>
<td></td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>(1.57)</td>
<td>(0.15)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*** Significant at 1 per cent *Significant at 15 per cent

4.34 This implies that the equity price gap affects IT and NICs receipts through wages and salaries; the equity price and transaction gaps affect VAT and excise duty through their effect on consumption, and the equity price gap moves financial sector corporate profits.

### Table 4.4: Asset price and transaction gap elasticities

<table>
<thead>
<tr>
<th>Tax Receipts</th>
<th>Equity price gap</th>
<th>House price gap</th>
<th>Transaction gap</th>
<th>Share of Total receipts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income tax and NICs</td>
<td>0.04</td>
<td></td>
<td></td>
<td>46%</td>
</tr>
<tr>
<td>VAT</td>
<td>0.04</td>
<td>0.02</td>
<td></td>
<td>15%</td>
</tr>
<tr>
<td>Excise duties</td>
<td>0.04</td>
<td></td>
<td>0.02</td>
<td>3%</td>
</tr>
<tr>
<td>Financial corporation tax</td>
<td>0.42</td>
<td></td>
<td></td>
<td>2%</td>
</tr>
<tr>
<td>Inheritance tax</td>
<td>0.45</td>
<td>0.80</td>
<td>0.02</td>
<td>1%</td>
</tr>
<tr>
<td>Capital gains tax</td>
<td>1.80</td>
<td>0.25</td>
<td>1.00</td>
<td>1%</td>
</tr>
<tr>
<td>Stamp duty land tax</td>
<td>1.20</td>
<td>1.00</td>
<td></td>
<td>1%</td>
</tr>
<tr>
<td>Stamp duty shares</td>
<td>1.00</td>
<td></td>
<td></td>
<td>1%</td>
</tr>
<tr>
<td>Total elasticity of receipts as share of GDP</td>
<td>0.02</td>
<td>0.01</td>
<td>0.01</td>
<td></td>
</tr>
</tbody>
</table>
Combining these estimates with the capital tax adjustments we find that the total tax elasticity to the equity price gap is around 0.02, to the house prices gap around 0.01 and to the transaction gap around 0.01. Despite using a very different approach the results are similar to the one-step approach using our preferred gap series, although the coefficient on transaction gap is smaller.

These estimates are consistent with the historical relationship between asset prices and transactions and relevant economic tax bases like financial sector profits. This could have changed in the wake of the crisis due to structural changes in the financial sector, for example related to the payments of financial sector bonuses.

Using the two-step approach asset market fluctuations are shown to have led to deviations of tax receipts from their structural level of up to around 0.6 per cent of GDP when wider taxes are included – Chart 4.4. In the run up to the financial crisis the gap is around 0.2 to 0.4 per cent of GDP largely driven by house prices. Again this is partly due to the fact that the equity price gap is estimated to have been relatively small in 2006-07 and 2007-08. Our results suggest a correction of around 0.4 per cent of GDP in 2008-09, due to the negative effect of the equity prices and transactions gaps on income tax and VAT receipts. This is, unlike the one-step results, somewhat offset by a positive house price gap boosting capital taxes. When we estimate the impact on capital taxes in the two-step approach we adjust the gap series for the cycle. The result is a larger positive house price gap following the financial crisis compared to the unadjusted series used in the one-step approach.
Chart 4.4: Adjustments to cyclically-adjusted tax receipts for asset price and transaction effects: two-step approach

A comparison with the result from the one-step approach is shown in Chart 4.5. The effect is smaller, which is mainly due to a smaller coefficient on the transaction gap in the two-step results. The difference is most pronounced in recent years, when the transaction gap is estimated to have been particularly large. This illustrates that not only can estimates of the asset price gap series differ considerably, leading to very different adjustments for tax receipts, but that small differences in the estimated relationship between asset price and property transactions and tax receipts also matter. This highlights the difficulty of using this approach for forecast purposes.
Asset price and transaction adjustments

Chart 4.5: Additional tax revenue from asset price and transaction disequilibrium: one-step and two-step results

Using the results from the two-step approach shown in Chart 4.4 we can show public sector net borrowing broken down by cyclical borrowing, borrowing linked to asset prices and transaction gaps and the residual, structural balance - Chart 4.6. In order to do this we re-estimated the cyclical adjusted PSNB using the results from Table 4.3 to avoid double counting the impact from equity prices and transactions on other taxes.
Comparing this cyclically and asset price and transaction gap adjusted PSNB in Chart 4.6 to cyclically adjusted PSNB from Chapter 2 we find the series to be relatively similar. Small asset price and transaction adjustments offset somewhat from lower cyclical adjustment. The deviations range from -0.4 to 0.4 per cent of GDP. In 2006-07 we find structural net borrowing corrected for asset price and transaction gap to be higher by around 0.4 per cent of GDP but around 0.3 per cent of GDP lower in 2008-09.

We could produce a similar chart using the results from the one-step approach which would suggest a larger adjustment for asset price and transaction gaps. This would lead to a larger deviation from the historical structural position shown in Chapter 2.

Conclusion

This analysis confirms that tax receipts are sensitive to changes in asset prices and transactions, as would be expected given the relative importance of the financial and housing sectors for UK tax receipts.

However there are conceptual problems with adjusting our cyclically-adjusted estimates for asset prices effects. The concept of an asset price gap is itself controversial, and this is reflected in the treatment of asset prices in the EFO forecast, which implicitly assumes that no such gap exists. This limits the scope for applying the analysis in this chapter to the EFO forecast.
In addition to the theoretical objections, the analysis has highlighted a number of practical problems in deriving equilibrium levels of asset prices. The two approaches we have used here can generate very different asset price gap series, and at several points neither series looks particularly plausible. In particular the different methods for calculating the equity price gap series give (unlike alternative estimates of the output gap) significantly different outcomes, which would suggest large differences in adjustments to receipts. This finding is common to other similar work produced by the OECD and the ECB, and there remains no consensus around the suitability and method of including asset prices in cyclical adjustment calculations.

For these reasons we do not think it suitable to attempt to adjust for asset price and transaction gap cycles in addition to the economic cycle when estimating the structural position of the public finances. However, the OBR could use this type of approach to undertake risk and sensitivity analysis in future EFO’s.

The gaps series can though be of practical use in understanding the historical relationship between asset prices, transactions and the output gap. This information was for example used to cyclically adjust capital taxes, in the two-step approach, in Chapter 2.
5 Conclusion

5.1 The specification of the Government’s fiscal mandate in cyclically adjusted terms requires the OBR to make an assessment of the effects of the economic cycle on the public finances. To date, the OBR has adopted the Treasury’s approach to cyclical adjustment as presented in the 2008 Treasury working paper: Public finances and the cycle. In this paper we revisit the Treasury analysis and consider other approaches including that used by the OECD. Estimating such coefficients involves a number of uncertainties so we have attempted to take a wide-ranging approach and have undertaken sensitivity analysis where possible.

5.2 We re-estimate the cyclical adjustment coefficients using both the Treasury’s approach and the two-step approach developed by the OECD. We find the two-step approach to have a number of advantages: it can be tailored to capture specific features of the UK tax and benefit system; it makes use of detailed and up-to-date UK tax and benefit forecast models; it does not require the construction of a policy adjusted-tax series; and the results are generally more robust econometrically.

5.3 Our central estimate from the two-step approach, after correcting for known lags in the tax system and extending the approach to capture a wider measure of cyclical spending, is a contemporaneous response of net borrowing as a share of GDP to the output gap of 0.5 and a one year lagged response of 0.2. This is the same result as was found using the Treasury approach in 2008. We also obtained the same results when we re-estimated the coefficients using the Treasury’s approach with the latest data.

5.4 We performed various sensitivity tests on our results for robustness. The results, for example, were not found to be very sensitive to different output gap series and were relatively stable over different economic cycles. The two-step results were also tested for different profit elasticities. These did not change the results significantly.

5.5 On this basis the OBR intends to continue to use coefficients of 0.5 and 0.2 to produce estimates of cyclically-adjusted net borrowing and the current budget in future editions of the Economic and fiscal outlook (EFO). While the approaches used all suggested similar sized coefficients we nevertheless recognise the significant uncertainties involved in the estimation process. The OBR will continue to show the sensitivity of its forecasts to alternative cyclical adjustment coefficients in the EFO.
Conclusion

5.6 In this paper we also consider the use of a structural VAR model to cyclically-adjust the public finances. This potentially addresses the issue that the output gap may be itself affected by fiscal policy, which could bias the econometric results of the OECD and Treasury approaches. However, the results are heavily dependent on the choice of modelling structure and assumptions and we do not believe it would be appropriate to use them in the EFO forecasts.

5.7 We estimated the cyclically adjusted current budget using the ECB approach which attempts to correct for changes in composition of demand. The results highlight the importance of the composition of GDP for the public finances. However the component gap series require an estimate of the benchmark composition of demand. But, unlike potential output, there is no equivalent reference for the equilibrium structure of aggregate demand. Furthermore the method is not as robust for forecast purposes since the simple gap estimates are less reliable at the end of the data series.

5.8 Fluctuations in asset prices and property transactions that are uncorrelated with the cycle can also affect tax revenues and potentially mask the true structural position of the public finances. In the paper we analyse the effect on UK tax receipts of deviations in equity, house prices and housing transactions from estimated equilibrium levels. The analysis suggests that asset prices and transactions may have affected the UK’s past fiscal position by as much as 1.3 per cent of GDP. But estimating an equilibrium path for asset prices is even more fraught with difficulty than estimating potential output. For this reason the OBR does not intend to incorporate an asset price adjustment directly into the cyclical adjustment methodology. However the broad approach in this paper could be used in future EFOs to show the sensitivity of the public finances to asset market movements, if it is judged to be a particular risk to the forecasts.

5.9 We re-estimate historical series for structural net borrowing using our estimated cyclical adjustment coefficients and output gap series. We found structural net borrowing to have been somewhat higher in the 1990s and in the run up to the 2008 financial crisis than previously estimated by HMT. Since the cyclical adjustment coefficients used are the same as used by the Treasury the difference is entirely explained by different estimates of the output gap.

5.10 We would welcome any comments or suggestions on the approach and results we set out here. Please email obrenquiries@obr.gsi.gov.uk.
6 References


Gottschalk, J., 2001, An introduction into SVAR methodology, identification, interpretation and intentions of SVAR models, Kieler Arbeitspapiere 1072


HM Treasury, 1995, Public finances and the cycle, The Stationery Office

HM Treasury, 1999, Public finances and the cycle, The Stationery Office

References


A Sensitivity and robustness

One-step method

A.1 For the one-step method we have tested the robustness of the results to two key sensitivities:

- the measure of the output gap used in the econometric regressions; and
- the sensitivity of the regression coefficients to the choice of sample period.

A.2 In our central estimates we use the output gap series published in the OBR’s Working paper No 1: *Estimating the UK’s historical output gap*. But the output gap is an unobserved variable and estimates of its size are therefore necessarily subjective and inherently uncertain. It is for this reason that the OBR subjects its forecasts for the fiscal mandate measure to sensitivity tests in Chapter 5 of the OBR’s *Economic and fiscal outlook*. Those calculations attempt to quantify the size of the error in the OBR’s assessment of the size of the current output gap that would likely cause the fiscal mandate to be missed.

A.3 However, while estimates of the current size of the output gap are contentious, there is typically more agreement about the time series profile of the output gap, as can be seen in Box 2.3. We might therefore expect the time series regression results to be relatively insensitive to the choice of output gap. To investigate this proposition we have repeated the regression analysis for the main equation in Chapter 2, but substituting the Treasury’s previous estimate of the output gap (which was last updated in the March Budget 2010) in place of our current estimate. Table A.1 compares the results for the coefficients of interest: those on the output gap.
Sensitivity and robustness

Table A.1: Sensitivity of the results to the choice of output gap

<table>
<thead>
<tr>
<th>Equation</th>
<th>Coefficient on output gap</th>
<th>Coefficient on lagged output gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate tax</td>
<td></td>
<td>0.14</td>
</tr>
<tr>
<td>TME</td>
<td>-0.44</td>
<td></td>
</tr>
<tr>
<td>C cyclical social security</td>
<td>-0.18</td>
<td></td>
</tr>
<tr>
<td>TME excluding CSS</td>
<td>-0.40</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation</th>
<th>Coefficient on output gap</th>
<th>Coefficient on lagged output gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate tax</td>
<td>0.23</td>
<td></td>
</tr>
<tr>
<td>TME</td>
<td>-0.48</td>
<td></td>
</tr>
<tr>
<td>C cyclical social security</td>
<td>-0.14</td>
<td></td>
</tr>
<tr>
<td>TME excluding CSS</td>
<td>-0.44</td>
<td></td>
</tr>
</tbody>
</table>

A.4 The results are very similar in most cases, with only the coefficient in the aggregate tax equation showing much of a departure from the estimates reported in Chapter 2. In general the estimates using the HMT output gap tend to report slightly larger coefficients, which can be explained by the slightly larger amplitude of the estimated cyclical variation in this measure.

A.5 The use of historical data means that the estimates reflect the average effect of changes in the output gap on the public finances over previous cycles. This means that if the current economic cycle differs from the average cycle, the relationship between the public finances and the output gap over the course of that cycle will not be captured in the coefficients. For example, the response of the labour market to the recent recession appears out of line with historical experience on the basis of the current vintage of data. The smaller rise in unemployment may imply a reduced sensitivity to the cycle of social security payments to that implied by the estimates of the average effect. On the receipts side, there is some suggestion that the recent large fall may reflect an increase in cyclical sensitivity (although it may be linked to the asset price effects discussed in Chapter 4).

A.6 We therefore use recursive and split-sample estimation to test the sensitivity of the regressions to the choice of sample period. For the recursive estimates we experiment with extending the sample both forwards (i.e. gradually expanding the sample period from 1972) and backwards (i.e. expanding the sample period backwards from 2010). The results for three of the main equations estimated in Chapter 2 are shown in Charts A.1 to A.3.
Chart A.1: Recursive estimation of the output gap coefficient: aggregate tax equation

Chart A.2: Recursive estimation of the output gap coefficient: TME excluding CSS equation
Sensitivity and robustness

Chart A.3: Recursive estimation of the output gap coefficient: cyclical social security equation

The results are quite mixed and point to some sensitivity in the results to the choice of sample period. In particular the results for the spending equation show a large discontinuity at the point at which the second time dummy is introduced in 2002 to account for the discretionary spending increase.

We also investigate dividing up our sample period into distinct economic cycles with the periods identified by fiscal years when the economy is thought to have been roughly at its potential level (i.e. a zero output gap). This is an attempt to see if the cyclical sensitivity of the public finances has evolved over time, without the potential bias introduced by sample periods in which the output gap does not average close to zero. However it inevitably does so with a considerable loss of sample size which severely limits the weight that can be placed on the results, which are summarised in Table A.2.
Sensitivity and robustness

Table A.2: Sensitivity of results to individual cycles

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OG</td>
<td>OG (-1)</td>
<td>OG</td>
<td>OG (-1)</td>
<td>OG</td>
</tr>
<tr>
<td>Aggregate tax eqn</td>
<td>0.14</td>
<td>0.11</td>
<td>-0.47***</td>
<td>0.52***</td>
<td>0.48*</td>
</tr>
<tr>
<td>TME equation</td>
<td>0.44***</td>
<td>0.37*</td>
<td>-0.81</td>
<td>-</td>
<td>0.88**</td>
</tr>
<tr>
<td>CSS equation</td>
<td>-</td>
<td>-</td>
<td>-0.17**</td>
<td>-0.31***</td>
<td>0.05</td>
</tr>
<tr>
<td>TME exc.</td>
<td>-</td>
<td>-</td>
<td>-0.71</td>
<td>-0.75</td>
<td>0.43</td>
</tr>
<tr>
<td>CSS eqn</td>
<td>0.40***</td>
<td>0.31*</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

*** Significant at 1 per cent; **significant at 5 per cent; *significant at 10 per cent.

A.9 In general the results are not robust to the reduction in sample size. The results for the 1974-08 period provide evidence that the use of larger sample periods, which includes incomplete cycles, does not significantly bias the result.

Two-step method

A.10 We also tested the results from the two-step method by analysing how the elasticity of tax and expenditure bases to the output gap varied over different economic cycles. We also looked at the relationship excluding the last two years.
Sensitivity and robustness

Table A.3: Elasticity of tax and expenditure base to the output gap, rolling cycles

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(OG)</td>
<td>(OG)</td>
<td>(OG)</td>
<td>(OG)</td>
<td>(OG)</td>
</tr>
<tr>
<td></td>
<td>(-1)</td>
<td>(-1)</td>
<td>(-1)</td>
<td>(-1)</td>
<td>(-1)</td>
</tr>
<tr>
<td>Wages and salaries</td>
<td>0.73***</td>
<td>0.83***</td>
<td>0.52</td>
<td>0.06</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>(0.20)†</td>
<td>(0.20)</td>
<td>(0.45)</td>
<td>(0.25)</td>
<td>(0.21)</td>
</tr>
<tr>
<td>Consumer expenditure</td>
<td>1.14***</td>
<td>1.17***</td>
<td>0.78*</td>
<td>0.77**</td>
<td>1.20***</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.13)</td>
<td>(0.37)</td>
<td>(0.25)</td>
<td>(0.20)</td>
</tr>
<tr>
<td>Non-oil, non-fin profits</td>
<td>4.16***</td>
<td>4.26***</td>
<td>7.06**</td>
<td>4.11***</td>
<td>1.83**</td>
</tr>
<tr>
<td></td>
<td>(0.60)</td>
<td>(0.63)</td>
<td>(2.90)</td>
<td>(0.67)</td>
<td>(0.69)</td>
</tr>
<tr>
<td>Fin profits</td>
<td>1.18*</td>
<td>1.46**</td>
<td>2.92</td>
<td>2.86*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.64)</td>
<td>(0.66)</td>
<td>(1.88)</td>
<td>(1.37)</td>
<td></td>
</tr>
<tr>
<td>Equity price</td>
<td>3.79</td>
<td>4.75**</td>
<td>0.58</td>
<td>8.15***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.60)</td>
<td>(2.16)</td>
<td>(2.89)</td>
<td>(1.60)</td>
<td></td>
</tr>
<tr>
<td>House price</td>
<td>3.48**</td>
<td>2.77</td>
<td>3.09</td>
<td>2.89**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.77)</td>
<td>(1.83)</td>
<td>(3.98)</td>
<td>(1.18)</td>
<td></td>
</tr>
<tr>
<td>Transaction gap</td>
<td>5.40**</td>
<td>6.85**</td>
<td>2.95***</td>
<td>7.95***</td>
<td>8.36</td>
</tr>
<tr>
<td></td>
<td>(3.00)</td>
<td>(2.99)</td>
<td>(0.93)</td>
<td>(2.11)</td>
<td>(6.78)</td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>-3.77***</td>
<td>-3.34***</td>
<td>-3.40***</td>
<td>-3.60***</td>
<td>-6.92***</td>
</tr>
<tr>
<td></td>
<td>(0.74)</td>
<td>(0.62)</td>
<td>(0.83)</td>
<td>(0.72)</td>
<td>(0.89)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table A.4: The budget balance semi-elasticity

<table>
<thead>
<tr>
<th></th>
<th>Year 1</th>
<th>Year 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net borrowing (1974-2008)</td>
<td>0.6</td>
<td>0.0</td>
</tr>
<tr>
<td>Net borrowing (1977-1986)</td>
<td>0.5</td>
<td>0.0</td>
</tr>
<tr>
<td>Net borrowing (1987-1997)</td>
<td>0.4</td>
<td>0.1</td>
</tr>
<tr>
<td>Net borrowing (1998-2008)</td>
<td>0.9</td>
<td>0.0</td>
</tr>
<tr>
<td>Net borrowing (All sample)</td>
<td>0.5</td>
<td>0.0</td>
</tr>
<tr>
<td>Net borrowing (All sample) with lags</td>
<td>0.5</td>
<td>0.1</td>
</tr>
</tbody>
</table>

A.11 For wages and salaries the coefficients are not significant in the cycle 1977 to 1986 and 1987 to 1997 but there is a strong contemporaneous effect in the economic cycle 1998 to 2008 suggesting that the relationship has changed somewhat over time. For financial company profits, equity prices and house prices the coefficient is also not significant in the economic cycle 1987-1997.

A.12 The effect on non-oil, non-financial profits diminishes over time and the time lag disappears. The relationship between the output gap, equity prices and transactions varies over time and the effect of the lagged output gap on unemployment is diminishing in the cycle 1998-2008.

A.13 Using the elasticities from Table A.3 we can estimate the total net borrowing or budget balance semi-elasticity, using historical averages for receipts and expenditure as a share of GDP over 2001-02 to 2010-11.
B The relationship between the one-step and two-step approaches

The relationship between the two methods

B.1 The two-step approach produces tax and expenditure elasticities that represent percentage changes as a share of potential output induced by a change in the output gap, equation (B1). Those elasticities are often referred to as sensitivity elasticities. The one-step approach on the other hand produces semi-elasticities that represent a change in the level of tax and expenditure as a share of actual GDP induced by a change in the output gap (equation (B1)).

\[ \frac{dT}{dY} = \eta_{T,y}, \quad \text{sensitivity:} \quad \frac{dT}{dY} \cdot \frac{Y}{T} = \varepsilon_{T,y} \quad (B1) \]

B.2 Equation (B2) shows how the sensitivity elasticity for tax receipts can be transformed to the semi-elasticity using the chain rule. A similar transformation works for expenditure.

\[ \text{Semi-elasticity: } \eta_{T,y} = \left( \frac{dT}{dY} \right) \frac{Y}{T} = \left( \frac{dT}{dY} - \frac{T}{Y} \right) * \left( \varepsilon_{T,y} - 1 \right) * \frac{T}{Y} \quad (B2) \]

B.3 The contemporaneous estimate of the tax sensitivity to potential GDP from the two-step approach was around 1.1 after allowing for clear lags in the system, see paragraph 2.72; the corresponding semi-elasticity would therefore be around 0.03, assuming tax receipts are around 37 per cent of GDP. Similarly the expenditure sensitivity to potential GDP was estimated to be around -0.04; the corresponding semi-elasticity would be close to 0.4.

B.4 The reason for the difference is that the semi-elasticity relates to the ratio of tax and expenditure to GDP while the sensitivity elasticity relates to the level of tax and expenditure, or tax and expenditure as a share of potential GDP. Any
The relationship between the one-step and two-step approaches

change in output is likely to be matched by a change in tax receipts leaving the tax to GDP ratio broadly unchanged, a semi-elasticity of around zero but suggesting a close to one-to-one change in the level of revenue to output a sensitivity elasticity of around one.

B.5 On the expenditure side, only unemployment related benefits are assumed to be cyclical and the other 99 per cent of expenditure is not. As discussed elsewhere, this is a reasonable assumption for much, but not of all outstanding components of expenditure. It does however suggest that a change in output is likely to have only a limited effect on the level of government expenditure, suggesting a sensitivity elasticity of close to zero, but some effect on the ratio of expenditure to GDP related to changes in the denominator.

B.6 To perform the same transformation on the lagged response (year 2) on receipts and expenditure the following formula applies:

\[ \text{Semi-elasticity: } \eta_{t,Y-1} = \left( \epsilon_{t,Y} \right) \frac{T}{Y} \]

(B3)

B.7 This suggests that the sensitivity elasticity from the two-step method is 0.1 in year 2 for tax and expenditure, after allowing for lags in the tax system.

Net borrowing and the cycle

B.8 Adjusting for the difference in definitions, the two approaches show similar outcomes for both tax and expenditure and therefore give broadly similar results when it comes to correcting net borrowing or the budget balance. The sensitivity elasticity, using the results from the two-step approach, tells us how much net borrowing would move in levels in response to changes in GDP - equation (B4). It is constructed as the sum of the two expenditure and tax sensitivity parameters weighed by their ratio to GDP. This elasticity should therefore be used when presenting the cyclical adjusted net borrowing or current budget as a share of potential output.

---

1 The two-step method attempts to estimate the following relationship (here X represents tax receipts or expenditure) \( \Delta \log(X_t/Y_t^*) = \beta + \alpha_t \Delta \log(Y_t/Y_t^*) + \alpha_t \Delta \log(Y_{t-1}/Y_{t-1}^*) \) (\( Y^* \) is potential output). If we subtract \( \Delta \log(Y_t/Y_t^*) \) from both sides we have \( \Delta \log(X_t/Y_t^*) = \beta + (\alpha_t - 1) \Delta \log(Y_t/Y_t^*) + \alpha_t \Delta \log(Y_{t-1}/Y_{t-1}^*) \) which is the estimate produced by the one-step method. This is a simple way to show how the two methods related when it comes to the lagged output gap impact, this is only consistent with variables expressed as a share of potential output.
The relationship between the one-step and two-step approaches

\[ \sigma_{b,y} : \left( \frac{dT}{dY} - \frac{dG}{dy} \right) \left( \frac{dG}{dY} \right) \left( \frac{dG}{dY} \right) \frac{T}{Y} - \left( \frac{dG}{dY} \right) = \epsilon_{t,y} \frac{T}{Y} - \epsilon_{g,y} \frac{G}{Y} \]  \hspace{1cm} (B4) 

B.9 The corresponding semi-elasticity, from the one-step approach, on the other hand would tell us how much net borrowing as a share of actual GDP would move in line with changes in GDP or the output gap.

\[ \tilde{\sigma}_{b,y} := \left( \frac{d}{dY} \frac{T}{Y} \right) - \left( \frac{d}{dY} \frac{G}{Y} \right) = (\epsilon_{t,y} - 1) \frac{T}{Y} - (\epsilon_{g,y} - 1) \frac{G}{Y} = \epsilon_{t,y} \frac{T}{Y} - \epsilon_{g,y} \frac{G}{Y} - \left( \frac{T - G}{Y} \right) \]  \hspace{1cm} (B5) 

B.10 The main difference is that a term for tax receipts minus expenditure as a share of GDP is included in equation (B5), which captures the effect of any change in GDP on the denominator of net borrowing or the current budget.

B.11 It is analytically more rigorous to present the fiscal aggregates as a share of potential output rather than actual output. This would correctly tell you what the ratio would be if the economy were at potential. However since most fiscal numbers are presented as a share of actual GDP the balance is rarely presented in that way.

B.12 Therefore it seems more appropriate to continue to use a semi-elasticity. The OECD, for example, uses the two-step approach to produce a sensitivity adjustment parameter before transforming to a semi-elasticity.
The relationship between the one-step and two-step approaches
C Simultaneous equation bias and the problem of identification

C.1 The impact of the output (gap) on the current budget can be estimated using the following simple OLS regression:

\[ b_t = \gamma_1 y_t + B_{bb}(L) b_t + B_{by}(L) y_t + e_{b,t} \]  

(C1)

where \( b_t \) is the current budget, \( y_t \) is output (or the output gap), \( B_{by}(L) y_t \) and \( B_{bb}(L) b_t \) are lags of output and the current budget respectively, and \( e_{b,t} \) the residual. The coefficient \( \gamma_1 \) can then be used to adjust the current budget for the effect of output.

C.2 Output enters equation (C1) contemporaneously and is in effect treated as an exogenous variable. However, this is unlikely to be the case in practice as output will also be driven in part by the current budget such that:

\[ y_t = \gamma_2 b_t + B_{yb}(L) b_t + B_{yy}(L) y_t + e_{y,t} \]  

(C2)

C.3 Output enters equation (C1) contemporaneously and is in effect treated as an exogenous variable. However, this is unlikely to be the case in practice as output will also be driven in part by the current budget such that:

\[ y_t = \gamma_2 b_t + B_{yb}(L) b_t + B_{yy}(L) y_t + e_{y,t} \]  

(C2)

C.4 If there is a significant feedback from the current budget to output \( (\gamma_2 < 0) \) simply estimating (C1) by OLS will yield a biased estimate of the coefficient \( \gamma_1 \) – that is the effect of output on the budget balance. This is because output will now be correlated with the residual in (C1) which contradicts the necessary condition for unbiased estimation\(^1\).

\(^1\) If \( \text{Cov}(y_t, e_{by}) < 0 \) implying that positive shocks to the budget balance (higher taxes or lower government expenditure) reduce output \( (\gamma_2 < 0) \) then \( \gamma_1 \) will be smaller than the true structural parameter and the effects of the economic cycle underestimated. The same issue would apply if we were estimating a fiscal impact model such as equation (C2) where \( \gamma_2 \) measures the impact of the current budget balance on output. Any positive feedback between output and the budget balance \( (\gamma_1 > 0) \) would lead to a biased coefficient if estimated by OLS such that the estimated coefficient \( \gamma_2 \) would be closer to zero than the true structural parameter.
Simultaneous equation bias and the problem of identification

C.5  One way of thinking about this potential bias is connected with the idea of identification. Clearly (C1) could easily be rearranged so that it has output on the left hand side and the current budget on the right hand side of the equation – in which case it would have the same functional form as (C2). Therefore, if (C1) is estimated by OLS it is unclear whether we are actually estimating a model of the cyclically-adjusted current budget or a fiscal impact model which shows the effect of the current budget on output. The coefficient of $\gamma_1$ is likely to be inconsistent reflecting an average of the underlying structural parameters $\gamma_1$ and $\gamma_2$ where the weights depend on the relative sizes of the structural disturbances $\epsilon_{t,1}$ and $\epsilon_{t,2}$. Because it is plausible that $\gamma_1 > 0$ and $\gamma_2 < 0$ then the average of coefficients will be lower than the true structural parameter of $\gamma_1$. This is the essence of simultaneous equations bias.

C.6  Identification could be achieved if we were able to find variables that affected output but were uncorrelated with the residuals in (C1). These variables would essentially act as instruments for output in (C1), which could be estimated using instrumental variable techniques such as two stage least squares (2SLS) and the generalised methods of moments (GMM) estimators to derive unbiased estimates of $\gamma_1$.^2

C.7  However, as Sims (1980) lays out in his famous critique of dynamic simultaneous equations models, it is difficult to find truly exogenous variables in macroeconomics to use as instrumental variables. For example, monetary policy variables may be considered to be a good instrument for output, but the current budget balance itself is partly determined by monetary policy variables such as interest rates on government bonds and Treasury bills.

^2 Murchison and Robbins (2003) use monetary policy variables, exchange rates and US GDP as instruments in their model of the Canadian cyclically adjusted budget balance. Their results show the cyclical adjustment coefficients are around twice as large under GMM (2SLS) than OLS.
D Structural VARs and identification

D.1 A vector auto-regressive (VAR) model is a system where each variable is regressed on its own lags and the lags of other variables in the system. An SVAR is a VAR model where a certain structure has been imposed so that the variables in the system are determined by the history of:

- structural shocks known as impulses; and
- the impact of these shocks on different variables in the system, which is characterised by an impulse response function.

D.2 One of the key elements of SVAR modelling is imposing a structure which gives these structural shocks economic meaning as well as impulse response functions that align with economic theory or empirical work.1 Because all variables in the system are treated as endogenously determined, an SVAR counters the Sims critique regarding the incredible identifying restrictions surrounding exogenous variables.

D.3 The starting point of the SVAR is to express the bivariate system in (C1) and (C2) in VAR form:

\[
\Gamma Y_t = B(L)Y_t + e_t, \quad (D1)
\]

where

\[
Y_t = \begin{bmatrix} b_t \\ y_t \end{bmatrix}, \Gamma = \begin{pmatrix} 1 & -\gamma_1 \\ -\gamma_2 & 1 \end{pmatrix}, B(L) = \begin{bmatrix} B_{bb}(L) & B_{by}(L) \\ B_{yb}(L) & B_{yy}(L) \end{bmatrix} \text{ and } e_t = \begin{bmatrix} e_{b,t} \\ e_{y,t} \end{bmatrix}
\]

D.4 The reduced form of (3) can then be estimated by OLS as follows

\[
Y_t = B^* (L)Y_t + u_t, \quad (D2)
\]

1 Gottschalk (2001) provides a useful survey of SVAR models and their interpretation.
Structural VARs and identification

D.5 where \( B^* = \Gamma^{-1}B \) are the reduced form coefficients and \( u_i = \Gamma^{-1}e_i \) are the reduced form residuals.

D.6 The next step is to convert the VAR into its moving average representation so the endogenous variables are expressed as a function of the reduced form innovations.

\[
Y_i = (I - B^*(L))^{-1}u_i \quad (D3a)
\]

\[
Y_i = C(L)u_i \quad (D3b)
\]

\[
\begin{bmatrix}
    b_i \\
    y_i
\end{bmatrix} = \begin{bmatrix}
    u_{b,i} \\
    u_{y,i}
\end{bmatrix} + \begin{bmatrix}
    C_{bb,1} & C_{by,1} \\
    C_{yb,1} & C_{yy,1}
\end{bmatrix} \begin{bmatrix}
    u_{b,i-1} \\
    u_{y,i-1}
\end{bmatrix} + \begin{bmatrix}
    C_{bb,2} & C_{by,2} \\
    C_{yb,2} & C_{yy,2}
\end{bmatrix} \begin{bmatrix}
    u_{b,i-2} \\
    u_{y,i-2}
\end{bmatrix} + \ldots \quad (D3c)
\]

D.7 This is the first representation of the impulse response function, showing how each variable is determined by a history of shocks. The only problem is that the system is not yet identified. We can trace through the impact of a shock on the economic variables in the system, but the shocks themselves have no economic meaning. The reduced form residuals are simply a linear combination of the underlying structural innovations.

D.8 Therefore the system needs to be decomposed so that the endogenous variables reflect the history of shocks to the structural innovations:

\[
Y_i = C(L)\Gamma^{-1}\Gamma u_i \quad (D4a)
\]

\[
Y_i = C^*(L)e_i \quad (D4b)
\]

D.9 where \( C^* = C(L)\Gamma^{-1} \) contains the impulse response to the structural innovations in \( e_i \) (\( e_i = (\Gamma u_i) \)).

D.10 However, identification will typically require a number of restrictions to be placed on the system. These generally take the following forms:

- Orthogonality: structural innovations are orthogonal meaning that they are uncorrelated \( \text{cov}(e_{b,i}, e_{y,i}) = 0 \). Bernanke (1986) describes structural innovations as primitive forces, not observed by econometricians, which buffet the system. This, however, does not mean that the endogenous variables in the system are not correlated; in fact the opposite is likely because they are driven by the same primary shocks;
Normalisation: it is common practice to standardise the system so that the impulse response functions are expressed in terms of unit innovations in the structural shocks; and

Identifying restrictions on $\Gamma$: there are various other types of restrictions that can be imposed on the system; these normally place short- or long-term restrictions on the response of endogenous variables in the system to the structural innovations. For example, Quinet and Bouthevillain (1999) impose the short-term elasticity of the budget balance to output in their SVAR model of the French cyclically-adjusted budget balance. Likewise, Blanchard and Quah (1989) impose long-run restrictions on the impact of demand and supply shocks in the economy.